TRACT II.

OBSERVATIONS

RESPECTING

THE TAKING DOWN AND REBUILDING

LONDON BRIDGE;

RELATIVE TO THE

PRINCIPLES OF BRIDGES:

WITH

TABLES OF BRIDGES, OF WEIGHTS AND MEASURES,
AND OF THE STRENGTH OF MATERIALS.

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RESPECTING THE TAKING DOWN AND REBUILDING

LONDON BRIDGE.

ON THE NEW BRIDGE.

A COMMITTEE of the House of Commons of this session, (1821,) recommends that the present London bridge should be taken down, and that a new bridge, of five arches, should be erected in the site of it. It is estimated to cost 450,000*l.*, including a temporary bridge, which is estimated at 20,000*l.*, but not including the compensations to parties injured, nor any consequent works from the alterations in the levels of the banks above London Bridge. The estimate is made from a comparison, magnitude for magnitude, with the cost of Waterloo bridge; and the sum of 50,000*l.* is added for contingencies. It is stated, that about 70,000 tons of stone will be required, and that the bridge is to be faced with Aberdeen granite, or granite of an equally hard and durable quality.

The marble bridge of Santa Trinita of Florence, so nobly rebuilt, " che non si è costrutto ponte piu bello dacchè si è rimessa la buona architettura," stands first in the scale or column of the specific height of the keystone in Table I. of bridges. If granite be as strong as marble, may we not have a bridge over the Thames something resembling the economy of figure and of material in this bridge, and not like the Egyptians, Greeks, and Romans, encumber the earth with a mass of stone large enough, according to the best guess which can be made to enclose the line of thrust, doubtful of its direction, and negligent of the expense of the structure? Ammanati was a countryman of Brunelleschi, the architect of the vault of Santa Maria del Fiore, of whom it has been said, "La posterita gli ha reso i dovuti onori; poichè in lui ha fissata l'epoca de risorgimento della buona architettura;" and both of them were employed in the Pitti palace. He was a countryman and contemporary of Michael Angelo and Galileo; in his time the investigations relating to the construction of and the building itself, of the dome of Santa Maria, were fresh in memory, and that of St. Peter was erected. In his time, the cycloid and the elastic and catenarian curves were invented, and the difficulty of investigating the properties of them may have driven, a few years afterwards, Newton, the Bernoullies, and Leibnitz to that invention; which, in the licence of poetry, has placed them in the same degree of approximation to angels as the ape is to man. The curve of the bridge of Ammanati is pointed, formed by the intersection of two arcs, probably of an elongated cycloid or of an elastic, logarithmic, or catenarian curve, the axis horizontal. Ferroni has shown that the curve is not an arc of an ellipse nor of a parabola, and, forgetting that the anse de panier is only an approximation to some other curve, and may be made to approximate to any, concludes that his approximation to the curve of the bridge is the curve itself. About the same time was introduced the arcuation adopted at King's college chapel, Cambridge, and Henry the Seventh's chapel, Westminster, of the same character; in the vaulting of which buildings architecture has obtained from geometry the last favour.

ON THE EFFECTS OF INCREASING THE WATER-WAY OF LONDON BRIDGE.

By a table of the levels of tides taken for a period of four weeks, it is manifest that if London bridge be removed, and what is called a proper water-way be given, that the ordinary high water of spring tides will sometimes be thirteen inches * higher above London bridge than at present; and, by the report of 1814, that it will be twenty-four inches higher; and it is probable that the extraordinary high tides may, in that case, rise three or four feet higher than the present extraordinary high tides. The Crown has considerable property in the low lands †; and that of individuals is to a great amount. Perhaps it may be thought prudent to determine the levels of the wharfs, river walls, and banks, and of the low lands westward of London bridge, on both sides the river, in respect to high tides, as far as the tide runs or may run in the case of the removal of the obstruction

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* See Rep. House of Com. 1821. But by other levels since taken, it will sometimes be 25½ inches instead of 13, and the average will be 12 instead of 8 inches; which confirms the report of 1814.

† The flood of the 28th Dec. ult. may be hereafter an ordinary occurrence.

to it at London bridge, and not trust to intuitive opinions, however eminent may be the persons who venture them, *before such a river, as the Thames, is let loose from a bondage of some centuries on a populous neighbourhood, in low lands, occupied like the theatre of the Philistines at Gaza, in the confidence of its enemy being shorn of his strength.* *

Mr. Foulds, engineer to London bridge waterworks, being asked, "Do you know what effect the enlargement of the centre arch had upon the influx and reflux of the tide?" answered, "It lowered the river so much upon the reflux, that craft could not navigate it in the accustomed manner." †

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* The crown, the city of London, the Dean and Chapter of Westminster, the Dukes of Northumberland and Devonshire, the Bishop of London, and other proprietors, can be paid for the injury they may sustain; but how those, *living* in the low lands, where the soil will be kept constantly saturated with water, and who will have the Thames, or rather the washings of the sewers, in their cellars and parlours, can be compensated, it is difficult to imagine. Sections of the Thames, between London and Blackfriars bridges, were in 1799 taken by order of the Trinity House. Similar sections ought to be taken, extending, in the lowlands adjoining, to a distance where the level of five feet above the present high water of ordinary spring tides, cuts the land as far up the river as the tide runs, before the bill for removing the dam of London bridge is introduced into Parliament.

† It is manifest from the Trinity House Sections, that if low water should be lowered the whole fall at London Bridge, the river eastward of Blackfriars Bridge to London Bridge will be reduced to a brook at low water of ordinary spring tides. And it may be concluded that between Westminster and Blackfriars bridges, where the river is very wide and shallow, it will assume the state of a ditch in an offensive swamp; for now, eight of the arches of Westminster bridge, five of Waterloo, and three of Blackfriars, are nearly dry at low water of spring tides.

See Proceedings and Evidence relative to London Bridge 1799 to 1819, and an Appendix 1820 to 1821, published by the city of London; and Reports of the House of Commons ordered to be printed 25th May, 1821, and 6th June, 1821; and Reports Improvement of London, May, 1796, and June, 1799.

At low water the exposure of stinking mud is now almost an intolerable nuisance to those living on the banks of the Thames above London bridge. Hereafter the whole bed of the river will be in some places nearly exposed.

This evil may lead to its remedy, and to a great improvement of the metropolis, viz. the taking from the river a width for a quay and street extending in length from Southwark bridge to the New Road at Millbank, and in width to low water; leaving the necessary docks accessible to coal and other barges under bridges excavated to such a depth below the level of low water, that the filth from the sewers may be at all times covered with water. The river is wide enough between Southwark bridge and Millbank to allow of the accession of this land to the metropolis; and the alteration would improve the navigation between those places, and relieve the thoroughfares of Cheapside, Fleet Street, and the Strand. The acquired ground would probably pay the expence.

Since writing the above, the author has read "Observations on the probable consequences of the demolition of London Bridge," by Sir Henry C. Englefield, in which he infers, from the different distances to which the spring and neap tides now flow, that the removal of London bridge would occasion the tide to flow about three miles higher than it does at present. He deduces, that the bridge considered as a bar, has become, from lapse of time, an essential part of the river; that the bridge prevents the tide from attaining so high a level above bridge as it otherwise would do; that it checks, in a considerable degree, the velocity of the flood-tide; that it prevents the tide from flowing so high up the country as it naturally would do; that the velocity of the reflux is in like manner checked; and that the water above bridge never ebbs out so low, by nearly the quantity of the whole fall, as it would do were the dam removed. "That any additional depth at high water would be perfectly useless to the navigation; that the water-way at the bridge at even common spring tides rapidly diminishes by the water rising in the curvature of the arches, and of course to a greater degree when the influx is very great. That the current at present is sufficient to carry, in one tide, craft from the pool to the extent of the up-current, and lighters have occasionally gone from Gravesend to Richmond in a tide; that an increased velocity would not be beneficial to the navigation, but on the contrary increase the hazard; that wherries and small craft would not be able to make way against the stream as they can at present; that any alteration which tends to give the waters of the river a quicker outfall, must injure the navigation; that the bed of the river near London will be nearly laid dry at the ebb of spring tides, and the filth from the sewers will have a much greater extent of shore to deposit itself on; that if the flood-tide ran stronger, the upper parts of the river would be choked with mud, carried up from London; and that less would be carried eastward than at present; for a more than ordinarily rapid current now causes a more than ordinary deposition of filth." He anticipates that the low lands from Rotherhithe to Battersea, including St. George's Fields, Vauxhall, and Lambeth, will be rendered uninhabitable or unhealthy from damps and stagnant waters. He refers to the fact, easily shown on a map of sewers, that the opposite shore of Westminster, from Privy Gardens to Ranelagh Gardens, was an island, and reasons on the injury the lower parts of the island may sustain. He anticipates injury to the low lands on each shore of the river as high as Kingston. *

* The Author has referred to the histories of the Thames and London Bridge, in order to reason from the past to the future. It appears that London Bridge was built by turning the course of the river; which confirms what may be deduced from the late probing of one of the piers, "That the bridge may stand upon piles, but *that it does not stand upon stilts.*" The prophesies of its sudden destruction rested upon *the stilts,* which have now given way under them. Before the bridge was a dam, the river was occasionally so dry, that people could walk over at low water; and that at high water, they could row their boats in Westminster, Lambeth, and other low lands adjoining the river. Since the erection of the waterworks, some of the marshes on each side the river have become towns, and islands have become peninsulas; by the removal of them, the towns may again become marshes, and the peninsulas may again become islands. † If the increased velocity given to the current, by removing the dam of London Bridge, should cause the bed of the river Thames to be proportionally deepened; the bridges west of London Bridge which have been erected with relation to the present bed of the river, which do not stand upon piles, may fall suddenly to ruin, by being undermined; and those which do stand upon piles may remain so much longer as the piles, then become stilts, retain their holds. It will therefore be prudent, on the part of the city, if London Bridge be taken down, to reserve the materials of the starlings and the old stone: the former may be wanted again for fences, and the latter like the stone of old Ludgate to stop a sudden demolition.

	London.	Southwark.	Blackfriars.	Waterloo.	Westminster.
‡ Superficial feet in waterways of the bridges, at the level of the Trinity high water mark, of	7360	13940	14117	17707	15198
‡ Superficial feet at the level of spring and neap tides, 14 feet 5 inches below ditto	1488	5012	3724	3382	3720

† The nine questions (see G. 5., App. page 138. 2d Rep. Lond. Port, 1796,) proposed in 1746, are not yet answered.

‡ App. Rep. II. of Com. Lond. Brid. 1821.

		London.	Southwark.	Blackfriars.	Waterloo.	Westminster.
* Linear feet of the collective spans of the arches, and widths of the bearing piers - -		927	708†	935	1240†	1068
* Linear feet of the collective spans of the arches		545	660†	788	1080†	860
‡ Descent of the road-way per yard in inches	north end	1¼		2⅜		3 1/10
	south end	1⅞		2½		2¾
§ Foot passengers	pass daily.	89640		61069		37820
Waggons		769		533		173
Carts and drays		2924		1502		963
Coaches		1240		990		1171
Gigs and taxed carts		485		500		569
Horses not drawing		764		522		615

The average time of high water London Bridge spring tides is 9 minutes earlier below than above bridge; of neap tides, 15 minutes.

Taken at Shadwell.	Feet	In.		H.	M.
	18	6	high water at	4	44
	18	0	¾ flood	3	33
	15	0	½ flood	2	22
	5	0	¼ flood	1	11
	0	0	low water mark	0	0

The tide rises 15 ft. out of 18 ft. 6 in. in half the time of the flood.

The stream at spring tides runs upwards about 20 minutes after it is high water by the shore, and in the middle from 30 to 32 minutes. 2d Rep. Lond. Port. App. C. 3.

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* App. BI. Rep. Lond. Port, Plate V. and MS. Drawings.

† By comparing the collective spans of the arches of Southwark and Waterloo Bridges; it appears that Waterloo Bridge might have had only five arches each 132 feet span; and 500 feet in length of that bridge might have been saved; and in that case, the whole of the inclined roads on the Surry side, parallel with the river, and the greater part of the compensation expence would have been saved also: for the descent would have terminated 500 feet nearer the river. The middle arch would then have been in the middle stream at low water nearly. A much greater saving even than this might have been made by following the example set at Verona. The excessive magnificence and strength of this bridge surely manifests the opulence of its founders as much as the great pyramid of Egypt does that of Cheops.

The cubical dimensions of the stone sunk to make the Breakwater at Plymouth, and of the stone, brick, and timber used at Sheerness, may bear a comparison with those of this pyramid. If we are as ostentatious of our wealth as the Egyptian, we apply it to works intended for the public benefit.

‡ App. B. 3. Rep. Lond. Port.

§ Month. Mag. March, 1816, and Morn. Chron. 26th May, 1812.

OBSERVATIONS

RELATIVE TO

THE PRINCIPLES OF BRIDGES.

In the general application of the properties of the common catenary to the equilibrium of an arch, it seems to have escaped observation that the chain forming the common catenary, if it be of a proper thickness at the points of suspension, in respect to tension at those points, must be too thick at every other point; for the tension is in the ratio of the secant of the angle made with a horizontal line, and the thickness is uniform; and if it be of a proper thickness at the apex, it will be too thin at every other point, and therefore fall to pieces; so, when inverted, and the tension is changed into compression or thrust, the thickness must be too great in every point with relation to the lowest points in the first case, and too little in the second; and as the connection which exists in a chain, and serves like a horizontal tie to maintain the catenarian direction of each link ceases when inverted, therefore an arch composed of equal lubricous spheres placed in the form of a catenary would not stand.

An investigation of the catenary, which is thick in every point, as the tension at that point, or as the secant of the angle made by that point with a horizontal line, and a mode of constructing it are given.

The thickness of the parts of an arch is determinable from the cohesiveness, in compression, of the material in respect to the curvature of the arch; in like manner, as the thickness of a chain is determinable from the cohesiveness, in tension, of the material in respect to the curvature of the catenary. Sir Christopher Wren proved, by reference to the columns in the naves of Gothic cathedrals, that weight would not compensate a want of due figure. Theorists of the present day, notwithstanding, maintain the contrary, and elicite that an abutment may have any thinness, if it be proportionably high. The principles which apply to an arch, apply to its abutment-piers *, for they are only the continuations of the cousinets, or they envelope a due elongated curve. †

The question to determine the extrados of an arch, and its abutment-piers, is of the nature of the problem, " To determine the locus of the extreme points of range of bodies projected in a horizontal direction

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* Dr. Hutton's Tracts, vol. i. p. 72., gives his proposition, " To find the thickness of the piers," &c. selected as he says, " among various ways, *only* as affording *probably a near* approach to a true conclusion." In the theoretical investigations of mathematicians in respect to abutments, the variable angle of thrust, which, when infants, cost them many a tumble to learn, is disregarded, and the law of our nurses to open our legs according to the direction and force of pressure substituted for another, namely, always to be provided with a great weight, so that, in case of being pushed we may increase our specific gravity.

† See Dr. Young's Nat. Phil. vol. i. p. 164. The author hopes that this very useful work will assume a more portable shape, and get into general circulation.

with a given horizontal velocity, from different points." * Let the bodies be so projected from points at heights each equal to the given horizontal velocity above any part of an ellipse, or any other curve, to determine the curve limiting the extreme points of range; the curve so determined is the extrados of the given arch, and of the abutment-piers by which it is elevated, assuming the thickness at the vertex for the horizontal velocity.

Early in his professional life, the author had occasion, for a particular purpose, to investigate the properties of arches and their abutment-piers, and published a treatise on the subject. His subsequent enquiries have only tended to confirm the principles which are there laid down. It was manifest to him that the figures of the buttresses of ecclesiastical buildings must have been obtained from some general principles, and not derived from the vague experience of the particular builder; and, from the great height of the vaults which they sustain, that those principles must be true, or be a close approximation to the true principles; and, being afraid of being deceived, in the same manner as he believed others had been deceived, by the demonstrations of mixed mathematics, he was led to take the greater number of the sections of ecclesiastical buildings contained in this work, and thought that if they confirmed what he had otherwise deduced, which they did in an extraordinary manner, he would be justified in the publication of a theory, though differing from those

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* This problem is investigated, in respect to the motion of projectiles from a point at different angles, in Gregory's Mechan. page 196.; in Bridge's Mechan. pages 197, 198.; and Whewell's Mechan. page 336.

which had been advanced by men of great eminence as mathematicians; and though, the work might be deficient in respect of the embellishment of the analytical investigations to determine the numerical values. The extrados which he deduced to the semicircular arch and its abutment-pier, drawn to a large scale, approximated so nearly, as apparently to coincide with the curve formed by a very fine gold chain suspended by its extremities in a vertical plane. He did not then investigate the curve of that extrados, nor distinguish it from the common catenary, as he ought to have done.

In order to show that information is wanted upon this subject, the author ventures to contrast the iron Menai bridge,—a case of tension, as intended to be erected according to the reports of the House of Commons, with the middle arch of the iron Southwark bridge, just completed, a case of compression, first comparing the tension of wrought iron with the compression of cast iron.*

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* By the experiments of Capt. Brown, Mill-Wall, Poplar, on a mean of eight experiments, "it requires 25 tons to tear asunder a piece of wrought iron, inch square, in direct tension." See Barlow's Essay, Strength and Stress of Materials, 1817, p. 235.

In the Appendix, page 28, to the Report from the Committee on Holyhead Roads, ordered to be printed 30th May, 1811, by the House of Commons, Mr. Telford is said to state, "from accurate experiments made by William Reynolds, of Colebrook Dale given to Mr. Telford for the Committee of the House of Commons, (See Appen. No. 13. Select Comm. Improv. Port of London, 1801,) that it requires 448,000 lbs. to crush a cube of $\frac{1}{4}$ of an inch of cast iron, of the quality called gun metal." But in the third Report Menai Bridge, ordered to be printed 29th April, 1819, pages 22, 26, and 30, Mr. Telford is said to state, that "from four to five tons *only* are required to crush a cube of one quarter of an inch, of moderately strong cast iron; multiplying this by 16 gives what a square inch will sustain." Professor

In the Menai bridge, the chord of the arch is 560 feet, the absciss, or versed sine, 37 feet; and there are to be only 16 cables, or bearers, of wrought iron, in a width of 30 feet, 4 inches diameter, weighing together 164 tons.

In Southwark bridge, the chord of the middle arch is only 240 feet, the versed sine 24 feet, and there are eight ribs or bearers of cast iron, in 45 feet, each 72 inches in height at the vertex, weighing together 907 tons. The radius of the circle of curvature at the vertex of the proposed pendent bridge over the Strait of Menai, is 1065½ feet; that of the middle arch of the Southwark bridge, at the vertex, 312 feet.

Then, by reference to the cohesiveness of cast iron in a state of compression, to wrought iron in a state of tension, to the radii of curvature at the vertices of the two bridges, it follows, that the one bridge is many hundred times too strong, or the other many hundred times too weak, if either of them be of a due strength.

If the true principles of bridges had been elicited, mathematicians would not continue the enquiry after them, nor would they be clogged by the modifications and conditions necessary to reconcile the discrepant results of different enquirers, and they would be

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[*] Robison, art. Strength of Mat. Encyc. Britt. 1797, says, that "the strength will increase in a greater ratio than the area of the section." Muschenbroek and Euler suppose that the strength of columns to be as the biquadrates of their diameters. Assuming that 25 tons are required to tear asunder, in direct tension, wrought iron inch-square in section, and upon "the well ascertained fact," according to the Report 1819, 70 tons may be taken as necessary to crush an inch square of cast iron.

confirmed by practice, and by the buildings of the times when arches were most in use and not condemned by them. When truth appears, she is manifest to the ordinary gaze; but at present the most eminent engineers seem to have obtained a very imperfect sight of her. For example: by reference to the tables of bridges, assuming the radius of curvature at the vertex a measure of the strength of a bridge in respect to figure; and the strength and weight of the material, and the incumbent weight, constant; it appears that Perronet, with ordinary stone, though so timid as to think it prudent to make the bridge of Neuilly * twelve times stronger than a breaking strength, performed more than is performed at the bridge of Conan †, in Ross-shire, in

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* It will be said that there are some settlements in the bridge of Neuilly; but they have little relation to the matter in question.

† The Report of the Commissioners for Roads and Bridges in the Highlands of Scotland has just fallen into the author's hands, in which there is an account of 1117 bridges, and drawings of 37. There is given the water-way of the bridges, but no relative waterway of the rivers; and, what is very extraordinary in the present state of knowledge, all the arches of which drawings are given are arcs of circles. The spans of the arches are given, but only in two instances are the versed sines. In no instance is the height of the key-stone given, nor the widths and heights of the abutments and bearing piers, nor the steepness of the roads over the bridges, except by a very small scale, which cannot be relied upon for such objects of inquiry. In this report the commissioners are made to say of their engineer, "that in all his plans and estimates of bridges and other masonry, he has manifested the most scrupulous economy, as never having been induced, by too much anxiety for his own character, to recommend needless solidity at the expense of his employers, we were obviously induced to secure his services," &c. This is a proper remark, if intended as a reproof to any one in particular, whose fears bias his judgment; but the application cannot be meant generally, which the consequence would warrant. In such a report, some disappointment must occur in not finding a demonstration of the fact. The

the ratio of 3.52 to 1; more than is performed at Waterloo bridge, though of granite, in the ratio of 2.22 to 1: and Ammanati performed more with marble than is performed at Waterloo bridge, with granite, as 22.2 to 8.0; more than is performed at the Southwark bridge with iron as 96 to 80; and than at Craigellachie in Morayshire, as 99 to 80.*

It is not probable that Edwards, at Pont y ty Prydd, learned his knowledge from Ammanati, but it is probable that Perronet derived some information from the work of Edwards. Taking into account the dates, Wales ranks before France in the art of bridge building, in respect to the due adjustment of strength to stress; and in respect to the theory and practice of bridge building generally, the most eminent mathematicians, engineers, and architects are less advanced than the Italians were two centuries ago, viz. in the time of Galileo and Ammanati.

In the Report of the Menai Bridge, ordered to be printed February 18th, 1819, Mr. Telford observes, in his Report, "It is only a few years ago that bridges of cast-iron, of one hundred feet span, were ventured upon with timidity; and now, although

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author believes that economy is the result of knowledge, and extravagance of the want of it. The architect of Santa Trinita recommended what he thought proper; and so doubtless did he of Alcantara, and therefore Ammanati may be accused, with as much propriety, of recommending needless solidity for selfish purposes, as the architect of the bridge of Alcantara. No engineer, called upon to act from his own knowledge, can be justified in putting to hazard his own character, by making a building less strong than he thinks it ought to be, unless his employers undertake the insurance of that character, as well as of the work at the same time endangered, or considered to be so.

* See the Tables of Bridges.

they have in some instances been mismanaged by bungling constructors, they have, in others, been successfully extended to 130, 140, and 240 feet arches, with scarcely any other approach to a maximum than what is determined by locality and expense." In comparing the tables of wooden and iron bridges together, it would seem that wood is a stronger material than iron; but as that is not the fact, it may be concluded, that the most successful constructors have yet a great deal to learn. Assuming iron and wood to be of the *same* strength in compression, the most successful constructors are deficient as 90 to 60. But taking the ratio of strength of iron to wood as the experiments give, then even Wiebeking may be more hardy in his opinions in respect to the proper height of the frame of an arch at the vertex of an iron bridge.

The Author, in constructing the tables of stone wood, and iron bridges, has referred to the following authorities; and where they differ, he has adopted those on whom, in his judgment, the greatest reliance could be placed.

Scelta di Archi. dal Ruggieri, edit. Bouchard, 1755.

La Verona illustrata, 1771.

Le Fabb. e, i diseg. di Palladio Scamozzi, 1778.

Mem. degli Archi. F. Milizia, 1781.

Le Antichità Romane. Piranesi, 1756.

Tomo 14 della Società Italiana delle Scienze del Ponte S. Trinita di Firenze. Ferroni.

L'Antiquité expliquée. Montfaucon, 1719.

Traité des Ponts. Gautier, 1728.

Cours d'Architecture. Patte's edit. Blondel, 1777.

Descrip. des Proj. et de la const. des Ponts. Perronet, 1788.

Antiquités de la France. Clerisseau, 1804.

Plans, coupes, &c. de l'Art de la Charpente Krafft, 1805.

Voyage Pitt. et Hist. de l'Espagne, 1806.

Traité de la const. des Ponts. Gauthey, 1809.

Gauthey refers to a German work, entitled, Historical Theatre of Bridges, by C. C. Schramn, and to a collection of drawings, belonging to M. Cretet, for French bridges; these works the author has not seen.

Traité Theo. et Pra. de l'Art de Bâtir. Rondelet, 1812.

Ogilby's China.

Plans, &c. by James Paine, 1767.

Vitruvius Britannicus.

Vitruvius Scoticus. W. Adam.

Wiebeking's Allgemeine theoretisch, praktishe, Wasserbaukunst, 1812.

Pope on Bridge Architecture, 1811.

Elem. Prin. of Carpen. Tredgold, 1820.

Edinburgh Encyclopædia, art. Bridge.

9th Report of House of Com. Highland Roads.

Fragmenta Vetusta. J. Halfpenny, 1807.

Sundry prints and private drawings.

FORMULÆ FOR CALCULATING THE DIAMETER, THE SEMISPAN, AND HEIGHT, OR VERSED SINE, OF A CIRCUCAR ARC.

Let the semispan = AC, and versed sine = CD, then $\frac{CD^2 + AC^2}{CD}$ = the diameter of the circle = $2r$.

$AC = \sqrt{(2r - CD)\,CD}$, and $CD = r - \sqrt{r^2 - AC^2}$

FORMULA FOR CALCULATING THE DIAMETER OF THE CIRCLE OF CURVATURE OF AN ELLIPSE AT THE VERTEX.

Let the transverse axis = AD, the semiconjugate = BC, then $\frac{AD^2}{2\,BC}$ = the diameter of the circle of curvature at the vertex.

Where the diameter of the circle of curvature at the vertex of a false ellipse has not been known, it has been calculated as in a true ellipse.

It may be observed from these tables, that it is the practice of architects to derive the ratio of the versed sine to the chord of the arch, from some regular figure inscribed in a circle; these ratios may be found from the following table. For example: To find the versed sine, or height of an arch of a bridge, when the chord or span of the arch is the side of an equilateral octagon. Let the chord be 100 feet, then by the 5th column .7654 will be the side of an equilateral octagon, the radius of the exscribing circle being 1, and by the same column the versed sine will be .0761 with the same radius.

Then as .7654 : 100 :: .0761 : 9.942 the versed sine required.

Let the versed sine, or height of the arch of the bridge be 9.942, required the chord of an arch of a bridge which is the side of an equilateral octagon. Then, as .0761 : 9.942 :: .7654 : 100, the chord required.

Let the chord be 100, and the side of an equilateral octagon. Required the radius of the exscribing circle, or arch of the bridge. Then, as .7654 : 1 :: 100 : 76.54 the radius required.

The Side of an Equilateral.

Triangle, or chord of 120°.	Square, or chord of 90°.	Pentagon, or chord of 72°.	Hexagon, or chord of 60°.	Octagon, or chord of 45°.	Decagon, or chord of 36°.	Dodecagon or chord of 30°.	Pentedecagon, or chord of 24°.
1.7320	1.4142	1.1756	1.0000	.7654	.6180	.5176	.4158

Semi Side of an Equilateral.

Triangle, or nat. sine of 60°.	Square, or nat. sine of 45°.	Pentagon, or nat. sine of 36°.	Hexagon, or nat. sine of 30°.	Octagon, or nat. sine of 22° 30′.	Decagon, or nat. sine of 18°.	Dodecagon or nat. sine of 15°.	Pentedecagon, or natural sine of 12°.
.8660	.7071	.5878	.5000	.3827	.3090	.2588	.2079

Versed Sine of the Arch * of a Bridge, when the Span of the Arch is the Side of an Equilateral.

Triangle, or of an arc of 60°.	Square, or of an arc of 45°.	Pentagon, or of an arc of 36°.	Hexagon, or of an arc of 30°.	Octagon, or of an arc of 22° 30′.	Decagon, or of an arc of 18°.	Dodecagon or of an arc of 15°.	Pentedecagon, or of an arc of 12°.
.5	.2929	.1910	.1340	.0761	.0489	.0341	.0219

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* In all cases the versed sine is to be understood to be the height of the arch from the middle of the chord or span.

Though an example of a cycloid is not given in the tables of bridges, the ratio of the versed sine to the span in the arches surbaisées most frequently adopted by the French and other architects, approaches very nearly to that of the cycloid, viz. As the diameter to the circumference of a circle, or 1 to 3.1416; and mathematicians have very properly recommended this curve in preference to the segment of a circle and to the ellipse "surbaissée au tiers," and its simplicity of construction, and the facility of finding the tangents (whence the joints of the voussoirs) warrants practically the recommendation.

Assuming the incumbrances on bridges to be a constant quantity, the radius of the circle of curvature at the vertex to be a measure of the strength of the form of the arch, the height of the key-stone to be determinable from the ability of the material to resist compression; and, that ordinary stone may be taken, after the proper allowances have been made to obtain a practical strength, of a constant strength in respect to compression; marble and granite, also, of a constant strength, but of greater strength, in that respect, than common stone; and brick of a constant strength, but of less strength than common stone; and so in respect to woods, and in respect to iron; then in the column of the specific height of the arch at the vertex, an *approximating* ratio may be had of the science displayed, of the quantity of materials used, and of the cost of the several bridges. For example, comparing the bridge of Neuilly, by Perronet, with that of Alcantara, over the Tagus, near Lisbon, Perronet's science will be to that of the architect of the bridge of Alcantara as 8.32 to 1., and the quantity of materials in the Alcantara bridge, and the cost of it will also be in the same ratio in respect to the bridge of Neuilly, viz. as 8.32 to 1.

TABLE I.—STONE AND BRICK BRIDGES.

Those marked M are Marble, those marked G are Granite, those marked B are Brick.

Name of the Bridge.	River.	Where.	Of the widest arch.								Specific height of the key-stone, the diameter of the circle of curvature at the vertex being 100.	Curve.	Architects.	Date of the erection.
			Length of the Chord or span.		Versed Sine or height.		Diam. of the circle of curvature at the vertex.		Height of the key-stone.					
			Feet.	In.	Feet.	In.	Feet.	In.	Feet.	In.				
Arch between the Towers of Lincoln Cathedral			28	0	0	9	262	0	0	11	.35	Seg. of circle		
Arch in the Church of the Jesuits at Nismes			28	3	0	4¼	565	0	2	1	.37	Seg. of circle	Cubisol	
Holy Trinity, M	Arno	Florence	95	3	14	10	345	3	2	9	.80	Pointed	B. Ammannati	1569
Neuilly	Seine	Paris	127	10	31	10	319 520	9 0	as designed 5 now	3	1.64 1.00	False ellipse	Perronet	1774
Nemours	Loing	Nemours	52	2	3	1	223	8	3	1	1.37	Seg. of circle	Boistard	1805
Ulm	Danube	Ulm	181	2	22	3	391	0	6	1	1.55	Seg. of circle	Wiebeking	1806
Pont y ty prydd	Taafe	Glamorganshire	140	0	35	0	175	0	3	0	1.71	Seg. of circle	Edwards	1755
Pont de la Concorde	Seine	Paris	93	9	9	8	237	0	4	2	1.75	Seg. of circle	Perronet	1791
Sainte Maixence	Oise	Sainte Maixence	76	8	6	3	240	0	4	8	1.94	Seg. of circle	Perronet	1784
Claix	Drac	Grenoble	150	2	62	3	154	3	3	1	1.99	Seg. of circle		1611
Avignon	Rhone	Avignon	110	9	45	10	112	5	2	3	2.00	Seg. of circle		1187
Boucherie	Pregnitz	Nuremburg	97	0	12	9	198	0	4	0	2.02	Seg. of circle	Pierre Carln	1599
Waterloo, G	Thames	London	120	0	32	0	225	0	5	0	2.22	Ellipse	Rennie	1816
Llanrwst	Conway	Denbighshire	58	0	17	0	67	0	1	6	2.24	Seg. of circle	Inigo Jones	1600
Jena	Seine	Paris	91	6	10	9	205	0	4	8	2.27	Seg. of circle	La Mandé	1815
Mantes	Seine	Mantes	127	10	38	3	268	10	6	3	2.32	False ellipse	Hupeau and Perronet	1765
Munich	Isar	Munich	102	3	17	0	170	0	4	2	2.46	Seg. of circle	Wiebeking	1814
	Garran	Inverary	60	0	15	0	120	0	3	0	2.50	False ellipse	Adam	
Vizile	Romanche	Briançon	137	4	38	3	246	6	6	3	2.53	False ellipse	Bouchet	
Gignac	L'Erault	Gignac	159	8	53	1	235	6	6	3	2.63	False ellipse	Garipuy	1793
Vieille Brioude	Allier	Brioude	183	3	70	3	195	3	5	3	2.68	Seg. of circle	Grenier and Estone	1454
Tongueland		Kircudbright	118	0	38	0	130	0	3	6	2.70	Seg. of circle	Telford	1806
Pesmes	Ougnon	Pesmes	44	8	3	10	131	7	3	10	2.91	Seg. of circle	Bertrand	1772
Richmond	Thames	Richmond	60	0	20	0	65	0	2	0	3.07	Seg. of circle	Paine and Couse	1761

TABLE I. — STONE AND BRICK BRIDGES — *continued.*

Name of the Bridge.	River.	Where.	Of the widest arch.								Specific height of the key-stone, the diameter of the circle of curvature at the vertex being 100.	Curve.	Architect.	Date of the erection.
			Length of the Chord or span.		Versed Sine or height.		Diam. of the circle of curvature at the vertex.		Height of the key-stone.					
			Feet	In.	Feet	In	Feet	In.	Feet	In.				
Carbonne, B	Garonne	Carbonne	102	3	39	3	133	1	4	2	3.13	False ellipse	Saget	1770
Dunkeld	Tay	Perthshire	90	0	30	0	97	0	3	2	3.27	Seg. of circle	Telford	1807
Bewdley			60	0	20	0	65	0	2	2	3.34	Seg. of circle	Telford	1809
Covered	Arno	Florence	98	0	17	3	156	4	5	3	3.35	Seg. of circle		
Conan	Conan	Ross-shire	65	0	21	0	71	0	2	6	3.52	Seg. of circle	Telford	1808
Ceret, B	Tech	Pratz de Mouillon	147	7	73	9	147	7	5	3	3.55	Semicircle		1336
Orleans	Loire	Orleans	106	7	29	9	190	9	6	10	3.58	False ellipse	Hupeau	1760
London	Thames	London	70	6	22	9	77	3	3	0	3.85	Seg. of circle	Dance and Taylor	1759
Henley	Thames	Henley	40	0	16	6	48	6	2	0	4.12	False ellipse	Hayward	1781
Des Têtes	Durance	Briançon	124	7	62	3	124	7	5	3	4.21	Semicircle	Henriana	1732
St. Esprit	Rhone	Languedoc	108	7	26	6	137	10	5	10	4.23	Seg. of circle		1305
Kew	Thames	Kew	60	0	30	0	60	0	2	7	4.33	Semicircle		
Lavaur	Agout	Lavaur	159	10	64	8	206	9	9	6	4.59	False ellipse	Saget	1775
	Garran	Inverary	60	0	20	0	65	0	3	0	4.61	Seg. of circle	Morris	
Tay	Tay	Scotland	60	0	20	0	65	0	3	0	4.61	Seg. of circle	Adam	
Atcham	Severn	Shrewsbury	50	0	25	0	50	0	2	6	5.00	Semicircle	Gwynn	1768
Ponte Cestio	Tiber	Rome	77	9	38	10½	77	9	4	3	5.46	Semicircle		380
Ponte Fabricio	Tiber	Rome	83	6	34	4	85	0	5	0	5.85	Seg. of circle		21
Blackfriars	Thames	London	100	0	41	6	112	0	6	7	5.88	False ellipse	Mylne	1771
Du Gard	Gard	Nismes	80	0	40	0	80	0	4	9	5.93	Semicircle		19 B.C.
Westminster	Thames	London	76	0	38	0	76	0	5	0	6.57	Semicircle	Labelye	1750
Ancient Bridge	Bacchiglione	Vicenza	34	10	11	0	37	3	2	9	7.38	Seg. of circle		
Alcantara, or The Bridge	Tagus	Lisbon	101	2	50	7	101	2	8	5	8.32	Semicircle		100
Augustus, M.	Luso	Rimini	28	4	14	2	28	4	2	9	9.70	Semicircle		

TABLE I. — STONE AND BRICK BRIDGES — *continued.*

Name of the Bridge.	River.	Where.	Of the widest arch.								Specific height of the key-stone, the diameter of the circle of curvature at the vertex being 100	Curve.	Architect.	Date of the erection.
			Length of the Chord or span.		Versed Sine or height.		Diam. of the circle of curvature at the vertex.		Height of the key-stone.					
			Feet	In.	Feet	In.	Feet	In.	Feet	In.				
Emissario	Del Lago	Albano	22„	8	11„	4	22„	8	2„	8	11.76	Semicircle		393 B.C.
Castel Vechio	Adige	Verona	159„	10	53„	3	239„	9				Elliptical		1354
Fouchards	Thouet	Saumur	85„	0	8„	7	219„	0				Seg. of circle	Limây	
Montlion	Durance	Montlion	102„	0	25„	6	204„	0				False ellipse	Delbergue-Cormont	1805
Toulouse, B.	Garonne	Toulouse	111„	9	39„	3	159„	0				False ellipse	Souffron	1632
Tournon	Doux	Tournon	156„	8	65„	0	159„	0						
Homps	Aude	Homps	70„	0	9„	4	140„	0				Seg. of circle	Ducros	1785
Rialto, M.	Canal	Venice	96„	10	20„	7	134„	4				Seg. of circle	Giovanni da Ponte	1591
Terni	Nera	Terni	131„	2	65„	7	131„	2				Semicircle		
Rumilly	Cheran	Rumilly	128„	0	64„	0	128„	0				Semicircle	Garella	1785
Vicenza	Bacchiglione	Vicenza	101„	2	29„	9	115„	8				Seg. of circle		
St. Diez	Meurthe		39„	0	3„	3	120„	0				Seg. of circle	Lecreulx	
Villeneuve d'Agen	Lot	Villeneuve d'Agen	115„	0	57„	6	115„	0				Semicircle		
Valence	Guadalaviar	Valence	42„	6	4„	3	111„	0				Seg. of circle		
Ouse Bridge	Ouse	York	81„	0	26„	3	88„	9				Seg. of circle		1235
Sisto	Tiber	Rome	83„	4	41„	8	83„	4				Semicircle		1474
Pont Mole	Tiber	Rome	77„	8	38„	10	77„	8				Semicircle		100 B.C.
Sisteron	Durance	Sisteron	85„	3	57„	4	63„	3				False ellipse		1500
Winston		Yorkshire	108„	0										1762
Sarah	Liffey	Dublin	106„	0	22„	0							Stevens	
Blenheim	Canal	Woodstock	101„	6								Seg. of circle	Vanburgh	
Bridge projected by Palladio			68„	0	22„	8	74„	0	3„	11	5.29	Seg. of circle		
Bridge projected by Perronet over the Seine at Melun			159„	10	14„	10	426„	4	5„	9	1.34	False ellipse		
Flying Bridge	Fi	Chogan China	600„	0										

TABLE II. — WOODEN BRIDGES.

Name of the Bridge.	River.	Where.	Of the widest arch.				Specific height of the arch at the vertex; the the diameter of the circle of curvature at the vertex being 100.	Curve.	Architect.	Date of the erection.
			Length of the Chord. or span.	Versed Sine or height.	Diam. of the circle of curvature at the vertex.	Height of the frame of the arch at the vertex.				
			Feet In.	Feet In.	Feet	Feet In.				
Tête		Picardy	125„ 8	6„ 3	638	3„10	.60	Seg. of circle	Coffinette	
Elsingen	Wertach	Elsingen	139„ 0	7„ 8	638	4„ 1	.64	Seg. of circle	Wiebeking	1809
Vilshoven	Vils	Vilshoven	179„ 0	11„ 7	703	4„ 9	.67	Seg. of circle	Wiebeking	1809
Bamberg	Regnitz	Germany	208„ 0	17„ 4	643	4„ 9	.73	Seg. of circle	Wiebeking	1809
Trenton	Delaware	Pennsylvania	200„ 0	32„ 0	345	2„ 8	.77	Seg. of circle	Burr	1804
Neuburg	Danube	Neuburg	156„ 9	14„ 0	453	3„ 9	.83	Seg. of circle	Wiebeking	
Freysingen	Isar	Bavaria	153„ 0	11„ 6	520	4„ 9	.91	Seg. of circle	Wiebeking	1807-8
Oettingen	Inn	Oettingen	103„ 6	7„ 2	381	3„ 7	.93	Seg. of circle	Wiebeking	1807
Pont Louis	Isar	Freysingen	154„ 0	13„ 6	452	4„ 7	1.01	Seg. of circle	Wiebeking	
Altenmarkt	Alz	Altenmarkt	143„ 0	13„ 6	392	4„ 7	1.16	Seg. of circle	Wiebeking	1809
Colossus	Schuylkill	Philadelphia	340„ 0	20„ 0	1465	17„ 0	1.16	Seg. of circle	Wernwag	1813
Rosen	Inn	Rosen	132„ 0	13„ 3	342	4„ 2	1.22	Seg. of circle	Wiebeking	
	Lech	near Augsburg	114„ 0	8„ 9	380	5„ 0	1.31	Seg. of circle	Wiebeking	1808
Muhldorff	Inn	Muhldorff	121„ 0	12„ 6	305	4„ 3	1.39	Seg. of circle	Wiebeking	
Tournus	Saone	Tournus	89„ 4	10„ 8	200	3„ 1	1.65	Seg. of circle	Gauthey	1801
Pont de la Cité	Seine	Paris	104„ 5	6„ 2	447	8„ 1	1.80	Seg. of circle	Dumoutier & Duvivier	1802
Mellingen	Reuss	Mellingen	157„ 6	20„ 6	323	5„10	1.80	Seg. of circle	Ritter	1794
Writtenghen		Switzerland	198„ 0	30„10	348	7„ 5	2.13	Seg. of circle	Grubermann	
Necker	Necker	Wirtemberg	63„10	9„ 6	116	3„ 1	2.65	Seg. of circle		
Piscataqua	Piscataqua	Near Portsmouth, North America	250„ 0	27„ 4	599	18„ 3	3.04	Seg. of circle	Palmer	1794
	Rott		94„ 9	19„ 0	139	4„ 6	3.23	Seg. of circle	Wiebeking	
Bridge designed by	Wiebeking		639„ 0	26„ 6	3876	4„ 2	.11	Seg. of circle		
Bridge designed by	Wiebeking		106„ 5	3„ 8	776	7„ 8	.98	Seg. of circle		
Bridge designed by	Palladio over	the Cismone	113„ 7	15„ 3	226	10„ 4	4.57	Seg. of circle		
Schuylkill	Schuylkill	Philadelphia	194„ 0	12„ 0	796				Palmer	1803
Deer's Island	Merrimack	Near Newbury Port	160„ 0						Palmer	1792

TABLE III. — IRON BRIDGES.

Name of the Bridge.	River.	Where.	Of the widest arch. Length of the Chord or span. Feet In.	Of the widest arch. Versed Sine or height. Feet In.	Of the widest arch. Diam. of the circle of the curvature at the vertex. Feet In.	Of the widest arch. Height of the frame of the arch at the vertex. Feet. In.	Specific height of the arch at the vertex; the diameter of the circle of curvature at the vertex being 100.	Curve.	Architect.	Date of erection.
Boston	Witham	Boston	85„ 0	5„ 6	332„ 0	3„ 0	.90	Seg. of circle	Rennie	
Southwark	Thames	London	240„ 0	24„ 0	624„ 0	6„ 0	.96	Seg. of circle	Rennie	1818
Sunderland	Wear	Sunderland	240„ 0	30„ 0	510„ 0	5„ 0	.98	Seg. of circle	Wilson	1796
Bonar	Arm of the sea	Sutherlandshire	150„ 0	20„ 0	301„ 0	3„ 0	.99	Seg. of circle	Telford	1811
Craigellachie	Spey	Morayshire	150„ 0	20„ 0	301„ 0	3„ 0	.99	Seg. of circle	Telford	1812
Bristol	Avon	Bristol	100„ 0	15„ 0	182„ 0	2„ 4	1.28	Seg. of circle	Jessop	
Austerlitz	Seine	Paris	106„ 0	10„ 6	278„ 0	4„10	1.73	Seg. of circle	Lamandé	1806
Buildwas	Severn	Buildwas	130„ 0	27„ 0	184„ 0	3„10	2.08	Seg. of circle	Telford	
Iron Bridge projected by Wiebeking			296„ 0	29„ 0	784„ 0	3„ 9	.47	Seg. of circle		

SUBSTANCE

OF THE

REPORT

FROM THE

SELECT COMMITTEE ON WEIGHTS AND MEASURES;

Ordered by the House of Commons to be printed; 28th May, 1821.[*]

THE Select Committee recommend :—

That the subdivisions of weights and measures employed in this country be retained, as being far better adapted to common practical purposes than the decimal scale.

That the parliamentary brass standard of three feet, now in the possession of the House of Commons, and made by Bird, in 1760, be henceforth considered as the authentic legal standard of length of the British empire, so that the distance between the centres of the two gold pins inserted in that scale, the brass being at the temperature of *sixty-two degrees* by Fahrenheit's thermometer, be one yard. And it appears from the experiments made for determining the length of the pendulum vibrating seconds at London, in a vacuum, and reduced to the level of the sea, that the distance from the axis of suspension to the centre of oscillation of such a pendulum, is 39,1393 inches of the above standard distance: and that the length of a platina metre at the temperature of *thirty-two degrees of* Fahrenheit's thermometer, supposed to be the ten millionth part of the quadrant of the meridian, corresponds with 39.3708 inches of the said distance.

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[*] Founded on the Third Report of the Commissioners appointed to consider the subject of weights and measures.

(Signed,) George Clerk,
Davies Gilbert,
Wm. H. Wollaston,
Thomas Young,
Henry Kater.

Ordered to be printed, 5th April, 1821.

The Appendix to the Second Report of the Commissioners contains a comparison of the customary measures employed throughout the country.[*]

That superficial measures remain as they are now defined by law, namely, that the perch, pole, or rod, be a square of 16½ feet, that the acre consist of 160 such perches, and so of the rest.

That the standard brass weight of two pounds, also in the possession of the House of Commons, and made in 1758, be considered as authentic; that one-half thereof, as gravitating in air at the mean height of the barometer, and with the thermometer at 62°, be henceforth the legal troy pound of the *British* empire, containing 5760 grains; and that 7000 grains troy be declared to constitute a pound avoirdupois.

And it appears that a cubic inch of distilled water weighs in a vacuum, opposed to brass weights in a vacuum also, at the temperature of 62° of Fahrenheit's thermometer, 252.72 such grains[*]; and consequently a cubic foot of distilled water, under similar circumstances, will weigh 62,386 pounds avoirdupois.

That the measures of capacity may be all brought back to an equality, and at the same time made to bear a simple relation to the standard of weight, by taking the pint for a basis, which contains 20 ounces of distilled water avoirdupois, at the temperature of 62°, as nearly as it is possible to ascertain by experiment, on a vessel of that construction and workmanship.

If, then, the pint be considered as equal in bulk to 20 ounces of distilled water, at the temperature of 62°, the cubic inch weighing 252.456 grains in air, at the mean height of the barometer, the imperial gallon will contain 277.276 cubic inches, weighing exactly *ten pounds.*

The following computations and proportions are contained in the *Appendix.*

The pendulum[†] vibrating seconds of mean solar time at London, in a vacuum, and reduced to the level of the sea, 39.1393 inches, consequently the descent of a heavy body from rest in one second of time, in a vacuum, will be 193.145 inches. The logarithm 2.2858828.

A platina metre at the temperature of 32°, supposed to be the ten millionth part of the quadrant of the meridian 39.3708 inches. The ratio to the imperial measure of three feet as 1.09363 to 1; the logarithm 0.0388717.

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[*] Or 252.456 in air, under the common circumstances of the atmosphere, when weights of brass are employed. In a vacuum at the maximum of density, that is, at 39°, the weight of a true cubic inch will be 253 grains, and of a cubic decimetre 15440. It appears, however, from an official report, obligingly communicated to us by Dr. Kelly, that the actual standard chiliogramme has been found to contain only 15438 English grains. The proposed imperial gallon of ten pounds, or 70000 grains of water, will contain very nearly 277.3 cubic inches, under common circumstances.

[†] See Phil. Trans. 1818, page 33. and Recueil D'Observations Géodésiques. Par Biot et Arago. Paris, 1821. page 587.

The five following standards have been measured as follows:

	Inches.
General Lambton's scale, used in the trigonometrical survey of India - -	35.99934
Sir George Shuckburgh's scale (which for all purposes may be considered as identical with the imperial standard) -	35.99998
Gen. Roy's scale - - -	36.00088
Royal Society standard - - -	36.00135
Ramsden's bar - - -	36.00249

	Logarithm.
Weight of a cubic inch of distilled water in a vacuum at the temperature 62°, as opposed to weights in a vacuum also, 252.722 grains - - - -	2.4026430
Consequently a cubic foot 62.3862 p. avoirdupois - - - -	1.7950887
Weight of a cubic inch of distilled water in air at 62° of temperature, with a mean height of the barometer, 252.456 grs. -	2.4021857
Consequently a cubic foot, 62.3206 p. avoirdupois - - - -	1.7946314
And an ounce of water, 1.73298 cubic inches - - - - -	0.2387924
Cubic inches in the imperial gallon, 277.276 - - - -	2.4429124
Diameter of the cylinder containing gallon at one inch high, 18.78933	1.2739112

SPECIFIC GRAVITY OF WATER AT DIFFERENT TEMPERATURES, THAT AT 62° BEING TAKEN AS UNITY.

70°	0.99913	56°	1.00050	44°	1.00107
68°	0.99936	54°	1.00064	42°	1.00111
66°	0.99958	52°	1.00076	40°	1.00113
64°	0.99980	50°	1.00087	38°	1.00113
62°	1.	48°	1.00095		
58°	1.00035	46°	1.00102		

The difference of temperatures between 62° and 39°, where water attains its greatest density, will vary the bulk of a gallon of water, rather less than the third of a cubic inch.

And assuming from the mean of numerous estimates, the expansion of brass 0.00001044 for each degree of Fahrenheit's thermometer, the difference of temperatures from 62° to 39°, will vary the content of a brass gallon measure just one-fifth of a cubic inch. It appears that the specific gravity of clear water from the Thames, exceeds that of distilled water at the mean temperature, in the proportion of 1.0006 to 1, making a difference of about one-sixth of a cubic inch on a gallon.

Rain water does not differ from distilled water, so as to require any allowance for common purposes.

TABLE OF THE MEASURES OF DIFFERENT NATIONS.

Foreign Measures compared with an English Foot.		Number of each equal to one English Foot.	Length of a single measure of each sort.
			English Inches.
Amsterdam,	foot	1.0762	11.15
Antwerp,	ditto	1.0676	11.24
Bavaria,	ditto	1.0508	11.42
Berlin,	ditto	.9844	12.19
Bologna,	ditto	.8005	14.99
Brussels,	ditto	1.0480	11.45
Copenhagen,	legal foot	.9717	12.35
Dantzic,	foot	1.0628	11.29
England,	ditto	1.	12.
Florence,	builder's braccio	.5555	21.60
France,	pied de roi	.9389	12.78
	toise	.1565	76.68
	metre	.3048	39.37
Geneva,	foot	.6250	19.20
Genoa,	palmo	1.2345	9.72
Hamburgh,	foot	1.0628	11.29
Lisbon,	ditto	.9278	12.96
Mantua,	braccio	.6575	18.25
Milan,	foot	.7682	15.62
Moscow,	ditto	.9112	13.17
Naples,	palmo	1.1562	10.38
Parma,	surveyor's braccio	.5623	21.34
Pavia,	ditto	.6557	18.30
Persia,	arish	.3136	38.27
Rimini,	braccio	.5610	21.39
Rome,	foot	1.0345	11.60
Russia,	ditto	.8727	13.75
Sardinia,	palmo	1.2270	9.78
Spain,	foot	1.0791	11.12
Sweden,	ditto	1.0266	11.69
Turin,	ditto	.9434	12.72
Venice,	ditto	.8772	13.40
Verona,	ditto	.8955	13.68
Vicenza,	ditto	.8804	13.63
Vienna,	ditto	.9639	12.45
Warsaw,	ditto	.8553	14.03

THE POUND OF DIFFERENT NATIONS.

	Corresponding number of English grains, Troy.	English Troy Weight.			
		lb.	oz.	dwt.	grs.
Amsterdam - -	7460.71	1	3	10	20.71
Antwerp and the Netherlands	7048.15	1	2	15	4.15
Copenhagen - -	6940.58	1	2	9	4.58
Dantzic - -	6573.86	1	1	13	21.86
Dublin - -	7774.11	1	3	19	18.11
England - -	5760, 0	1	0	0	0
Florence - -	5286.65	0	11	0	6.65
Genoa - -	4426.05	0	9	4	10.05
	6637.85	1	1	16	3.85
Geneva - -	8407.45	1	5	10	7.45
Hamburgh - -	7314.68	1	3	4	18.68
Lisbon - -	7005.39	1	2	11	21.39
Madrid - -	6544.33	1	1	12	16.33
Naples - -	4951.93	0	10	6	7.93
Paris - -	7560.80	1	3	15	0. 8
Rome - -	5257.12	0	10	19	1.12
Stockholm - -	9211.45	1	7	3	19.45
Turin and all Piedmont	4939.62	0	10	5	19.02
Venice, greater pound	4215.21	0	8	15	15.21
Venice, lesser pound	6826.54	1	2	4	10.54
Verona - -	5374.44	0	11	3	22.44
Vicenza - -	4676.28	0	9	14	20.28
	6879.05	1	2	6	15.05

⁂ To convert the above Troy grains into English avoirdupois pounds, divide by 7000.

See Mr. Barlow's New Math. Tables, 1814, among which will be found very extensive tables of the weights and measures of different countries.

TABLE OF THE STRENGTH OF MATERIALS.*

COMPRESSION.			
Material.	Weight of a cubic foot in ounces.	lbs. avoirdupois required to crush a cubic quarter of an inch.	Modulus of fracture or height of a column one quarter of an inch square in section of the material, equal in weight to the crushing weight. Feet.
Cast iron taken from the block - - - - (Each from an average of four experiments.)	7033	9773.5	51229
Ditto (horizontal casting)	7113	10114	52422
Ditto (vertical ditto) -	7074	11136.75†	58040
Cast copper crumbled with -	8788	7318	30698
Fine yellow brass - - -	8396	10304	45246
Wrought copper - -	8879	6440	26737
Cast tin - - -	7291	966	4884
Cast lead - - -	11352	483	1568
		lbs. avoirdupois to crush a cubic inch.	of a column inch square.
Elm - - - -	500	1200	5530
Fir - - - - -	600	2000	7680
English oak - - -	845	4000	10906

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* See Experiments, by George Rennie, Jun. Esq., Part I. Phil. Trans. 1818. The table, page 24, Tract III. was printed before the author had seen this paper. See Barlow's Essay, Strength and Stress of Timber.

† Mr. Reynolds deduced that 448000 lbs. avoirdupois, or 200 tons, were required to crush a cubic quarter of an inch of iron, cast at the furnace of Maidley Wood, of the quality called gun-metal; whereas these experiments show that five tons only are required, and one of them was upon gun-metal. The strength of the cast iron bridge of 600 feet span, projected by Messrs. Telford and Douglas, for London Bridge, was derived from Mr. Reynolds' experiment; and so late as 22d April, 1811, it was considered accurate. Probably all iron bridges constructed before Mr. G. Rennie's experiment in 1818, are founded upon data erroneous in respect to the strength of cast iron, in the ratio of 200 to 5. See Appen. 13 Lond. Port. sel. com. 1801, and Mr. Telford's Rep. to Lords of Treasury, 22d April, 1811. App. Rep. Holyhead Roads, page 28.

See Lecture 13th, Course of Lec. on Nat. Phil. Vol. I.; and Table, Vol. II. page 509. Dr. Thomas Young, M.D.

COMPRESSION.

Material.	Weight of a cubic foot in ounces.	lbs. avoirdupois to crush cubes of one and a half inch.	Modulus of fracture or height of a column inch and half square in section of the material equal in weight to the crushing weight. Feet.
Chalk	2500	1127	461
Brick of a pale red colour	2085	1265	621
Red brick, mean of two trials	2168	1817	858
Yellow face baked Hammersmith paviers, 3 times		2254	1064
Burnt ditto, mean of two trials		3243	1532
Stourbridge or fire brick		3864	1825
Derby grit, a red friable sand-stone	2316	7070	3126
Ditto, from another quarry	2428	9776	4123
Killaly white freestone, not stratified	2423	10264	4338
Portland*	2428	10284	4337
Craig Leith, white freestone	2452	12346	5156
Yorkshire paving, with the strata	2507	12856	5251
Ditto ditto, against the strata	2507	12856	5251
White statuary marble, not veined	2760	13632	5058
Bramly Fall sand-stone, near Leeds, with strata	2506	13632	5571
Ditto, against the strata	2506	13632	5571
Cornish granite	2662	14302	5502
Dundee sand stone or Brescia, two kinds	2530	14918	6038
Craig Leith, with the strata	2452	15560	6498
Devonshire red marble, variegated		16712	6623
Compact limestone	2584	17354	6878
Peterhead granite, hard close grained		18636	7386
Black compact limestone, Limerick	2598	19924	7607
Purbeck	2599	20610	8120
Black Brabant marble	2697	20742	7876
Very hard freestone	2528	21254	8610
White Italian veined marble †	2726	21783	8183
Aberdeen granite, blue kind	2625	24556	9581

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* To crush a two-inch cube of Portland required 14918 lbs.

† Rondelet's experiments shew statuary marble to be stronger than veined marble in the ratio of 8176 to 7455.

TENSION.

Material.	Weight of a cubic foot in ounces.	lbs. avoirdupois required to draw asunder a bar six inches long ¼ of an inch square.	Modulus of fracture or length of a rod one quarter of an inch square in section of the material equal in weight to the tensile weight. Feet.
Cast iron bar, horizontal - -	7113	1166	6043
Ditto vertical -	7074	1218	6347
Hard gun-metal, mean of two trials - - -	7033	2273	11914
Cast steel previously tilted -	7113	8391	43492
Cast copper - - -	8788	1192	5000
Fine yellow brass - - -	8396	1123	4931
Cast tin - - -	7291	296	1497
Cast lead - - -	11352	114	370
English iron reduced per hammer	7788	3492	16529
Swedish iron ditto ditto -		4504	21319
Shear steel ditto ditto -	7840	7977	37508
Blister steel ditto ditto -		8322	39131
Wrought copper ditto ditto -	8879	2112	8769

Rondelet, page 208. Vol. I. has given a table, in which is indicated the specific gravity, weight per foot cube, and weight required to crush a cube of 25 centemetres base, and one 4 inches base of 180 different kinds of stone, and of 18 different kinds of Basaltes, porphyries, granites, and different kinds of marbles.

He observes, that the heaviest stones are not the strongest. With an equal or less specific gravity, those of the finest grain, most compact texture, and deepest colours, will support the greatest weight. The strength of ordinary stones of the same kind, colour, and grain, augments with the specific gravity. He says, page 97. Vol. III. All these experiments prove that the strength of stone, of the same nature and form, increases very nearly in the ratio of the area of its base. Gauthey, Vol. I. page 275 and 276, says, the results reported by M. Rondelet, agree in general with mine, (see table 4, Gauthey, Vol. I.) and that it is convenient and safe in the application to assume, that it does so increase.

Mr. George Rennie, page 132. Phil. Trans. 1818. remarks, "In observing the results presented by the preceding table, it will be seen that little dependence can be placed on the specific gravities of stones, so far as regards their repulsive powers, although their increase is certainly in favour of their specific gravities. But there would appear to be some undefined law in the connection of bodies with which the specific gravity has little to do. Thus statuary marble has a specific gravity above Aberdeen granite, yet a repulsive power not much above half the latter. Again hardness is not altogether a characteristic of strength, inasmuch as the limestones which yield readily to the scratch, have nevertheless a repulsive power approaching to granite itself.

It is a curious fact, in the rupture of amorphous stones, that pyramids

TRANSVERSELY.						
Material—Cast iron.	Weight of the bars.		Distance of bearings, ends loose.		lbs. avoirdupois required to break the bars described in the first column.	Modulus of fracture or length of a rod inch square in section of the material equal in weight to the breaking weight. Feet.
	lbs.	oz.	feet	inches		
Bar of one inch square	10	6	3	0	897	259
Ditto ditto	9	8	2	8	1086	304
Half the above bar			1	4	2320	651
Bar of one inch square through the diagonal	9	8	2	8	851	238
Half the above bar			1	4	1587	445
Bar of 2 inches deep by ½ inch thick on edge	9	5	2	8	2185	624
Half the above bar on edge			1	4	4508	1290
Bar of 3 inches deep by ⅓ inch thick on edge	9	15	2	8	3588	962
Half the bar, ditto			1	4	6854	1839
Bar 4 inches by ¼ inch thick do.	9	7	2	8	3979	1125
Equilateral triangles, inch area in section — angle up	9	11	2	8	1437	395
Equilateral triangles, inch area in section — angle down	9	7	2	8	840	237
Equilateral triangles, inch area in section — half the first bar			1	4	3059	842
Equilateral triangles, inch area in section — half the second bar			1	4	1656	467
An inch bar, the end made fast			2	8	1173	
An inch square bar bore	One end made fast, the weight being suspended at the other at 2 feet 8 inches from the bearing				283	
A bar 2 inches deep by ½ an inch thick on edge					539	

are formed, having for their base the upper side of the cube next the lever, the action of which displaces the sides of the cubes, precisely as if a wedge had operated between them. I have preserved a number of the specimens, the sides of which, if continued, might cut the cubes in the direction of their diagonals."

By referring to the column of the modulus of fracture, which is also a measure of the compression at the vertex, or horizontal thrust of an arch, or the limit of the radius curvature at that point, it will be seen, that an arch bearing nothing but its own weight of cast iron (vertical casting) is limited to a radius of curvature at the vertex 58040 feet in length = 4 miles nearly; of English oak, the radius of curvature at the vertex is limited to 10906 feet = $2\frac{1}{15}$ miles; of Aberdeen granite, the radius of curvature is limited to 9588 feet = $1\frac{9}{11}$ miles. It will be observed from this, that the radius of curvature of an arch of oak supporting only its own weight, is greater than that of granite; but it must not therefore be concluded that a less quantity of oak will be required for an arch cæteris paribus than of granite, when a weight is to be borne. See Table of Examples.

APPLICATION OF THE FORMULÆ DERIVED IN TRACT 3, FROM THE INQUIRY INTO THE PRINCIPLES OF PENDENT BRIDGES, TO DETERMINE THE PROPORTIONS OF THE PARTS OF INSISTENT BRIDGES.

In like manner as the length of a chain of an uniform thickness *hanging* vertically, is a measure of the tension at its upper extremity (see Tract 3. Art. 6. page 103), so the length of a column of an uniform area in section *standing* vertically will be a measure of the compression at its base.

By a parity of reasoning from the article page 84, Tract 3. and the subsequent theoretical investigation of the pendent catenary, it may be derived in respect to insistent arches:

That the force of compression at the vertex of an insistent arch, equally stable in all its parts, and in the same or horizontal direction every where, is equal to that of the weight of a prismatick column of the same material as the arch is composed of, having its section equal in area to a section of the key stone in the direction of the radius of curvature at the vertex, and its height equal to that radius of curvature; and supposing the repulsive strength of homogeneous materials to vary as the area of the section; the lengths of the joints of the voussoirs of such an arch at right angles thereto must vary as the secant, of the angle the curve makes with a horizontal line, or of the angle made by the joint with a vertical one: and it also may be derived, that such an insistent arch may be produced to any distance from the vertex without altering the compression of any given joint.

Feet.

Let the radius of curvature at the vertex - - $= c$
the height of the key stone - - - $= n$
the thickness at any other voussoir - - $= l$
the breadth of the arch from face to face - - $= b$
the semi-span of an arch - - - - $= y$
the versed sine or height of an arch - - $= x$

lbs. avoird.

the weight of one cubic foot of the materials composing the arch - - - $= g$
the weight of a prismatick column one foot square of the given materials standing vertically, necessary to cause fracture at its base $= f$
the weight of a prismatick column of the same material, having its section equal to one square foot, and equal in weight to the weight to be placed upon or to be borne by each superficial foot at the vertex - - $= w$
the angle which any joint makes with a horizontal line $= \varphi$

Since it is shown in Tract 3. page 85, that $(ng + w)\, t = (ng + w)\, c \sec. \varphi =$ the tension at the point of suspension when the breadth is considered as $= 1$; by multiplying by b, we have the like tension when the breadth is $b = (ng + w)\, bc \sec. \varphi$, hence changing tension into compression, and referring this expression to the vertex where $\varphi = 0$, or $\sec. \varphi = 1$, we have the absolute force of compression at the vertex of such an arch, taking into consideration the effect of incumbent weight, $= (ng + w)\, bc$.

Also because the compression at the vertex is always equal to the force of horizontal thrust, we have the force of the horizontal thrust at any point in such an arch, taking into consideration the effect of incumbent weight $= (ng + w)\, bc$.

Again, if we suppose the arch not more liable to be crushed at any other point than at the vertex, we may, from the above expression, determine the height of the key stone.

For since the height of the key stone $= n$ feet, and the breadth of the arch from face to face $= b$ feet, we have the area of the section at the vertex $= nb$ square feet. And since the area of the vertical section $= nb$ square feet and the repulsive strength of the materials $= f$ in lbs. per square foot, it follows by hypothesis, that nbf must represent the force which would be just sufficient to crush the key stone, whence $nbf = (ng + w)\ bc$ or by dividing each side by b; $nf = (ng + w)\ c = ngc + cw$, from which again

$$nf - ngc = cw \text{ or } n = \frac{cw}{f - cg} = \frac{w}{\frac{f}{c} - g}*$$

$$l = n \sec. \phi$$

$$\text{Also } w = n\left(\frac{f}{c} - g\right)$$

$$\text{And } c = \frac{nf}{ng + w} = \frac{f}{g + \frac{w}{n}}$$

and if in this last expression we make $w = o$, we have $c = \frac{f}{g}$.

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* If in the formula $n = \frac{w}{\frac{f}{c} - g}$ we divide, both the numerator and denominator by g and put $\frac{w}{g} = w'$ and $\frac{f}{g} = f'$ we shall have f' from the table of the strength of materials under the heading "Modulus of Fracture," and g may also be obtained by the same table, under the heading "Weight of a Cubic Foot," and w is an arbitrary quantity depending upon the weight to be borne by each superficial foot at the vertex, and the excess which the architect may think necessary to add for contingencies, from which w' may be found by the formula $w' = \frac{w}{g}$ and in that case

If we suppose circles touching one another at the vertex were described with the different radii due to different materials obtained from the formula $c = \frac{f}{g + \frac{w}{n}}$ and arches cut off by the same span or the same versed sine, the specific heights or versed sines and specific semi-spans of a circular arch will be obtained from the common formulæ.

$$x = c - \sqrt{c^2 - y^2}$$
$$y = \sqrt{(2c - x)\,x}$$

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all the dimensions in feet, and $n = \frac{w'}{\frac{f'}{c} - 1}$, $l = n$ sec. ϕ.

$w' = n\left(\frac{f'}{c} - 1\right)$ and $c = \frac{f'}{1 + \frac{w'}{n}}$

M. de Prony, in his Nouvelle Hydraulique, vol. i. de l'équilibre des voutes, art. 373. deduces a formula to determine the height of the key of an arch, and one to determine (art. 375.) the greatest span with relation to the strength of the stone. He observes, when experiments showing the strength of stones are given and applications of the formulæ made; it will be seen with astonishment how much more hardy the results of calculation give the dimensions, than those which are used in practice. He proposes to discuss at a future time the modifications which physical causes and the obstacles of construction will render necessary in such results, and to elicite the limits of hardiness possible in construction, or the bounds which nature has assigned to art. M. Garnier has annexed to the commencement of the second volume, by the consent of M. de Prony, what he calls an éclaircissement to art. 373; and from the formulæ has calculated a table, the data drawn from the Bridge of Neuilly. This table makes the absolute pressure per foot square on the joint at the vertex, equivalent to 17612 French lbs. (or a column of the stone only * $\frac{17612}{152} = 115{,}9$ feet high of that base, and $\frac{1580}{115{,}9} = 14$ nearly) or $\frac{1}{14}$ of the modulus of fracture, the same as deduced by M. Gauthe.

* Note, see radius of curvature and weight per foot cube, page 45. of this Tract, and art. 373 of M. de Prony's work.

OF THE LIMIT OF WEIGHT WHICH A BRIDGE OUGHT TO BE ABLE TO SUSTAIN, OF THE PROBABLE WEIGHT, AND INSURANCE WEIGHT.

The moving force on an insistent bridge may be taken at 40lbs. per foot superficial, and to move at the rate of about four miles per hour, or 6 feet per second; its motion will cause percussion when it meets with obstacles, and therefore at that rate may be taken at 55lbs.; the puddling or cementitious covering upon the extrados, and the gravel and matter to form the roadway at 195lbs. more, together 250lbs. avoirdupoise, on stone bridges. On wooden bridges, 150lbs. may be taken.

Rondelet, page 101. vol. iii. says, from all the experiments given by him and by an infinite number of others, that stone begins to split under a pressure of a little more than half the weight which would crush it. Gauthey says, page 272, "consequently there cannot be any danger in making stone bear one-third of the weight which would crush it." Gauthey, page 274, vol. I. says, that the weight supported by the stone of Bagneux, in the pillars of the dome of St. Géneviève, is between 8 and 9 times less than would crush it. He says, page 275, that the key stone of the bridge of Neuilly supports a weight 14 times less than would crush the stone. By the calculation given, it would be crushed by an incumbent weight per square foot of 3966lbs. and it has been shown that 250lbs. may be taken for the probable incumbent weight, $3966 - 250 = 3716$ and $\frac{3716}{250} =$ not quite 15, that is, 15 times more weight than it is probable will ever come upon it, will be necessary to crush it. Perronet says, the piers sustain 12 times less than would crush the stone, page 626 of his work. Perronet was a man of experience and science, and knew by experiment the strength of the stone which he used; it may not be prudent at present to be more hardy than he was.

The discretion and experience of the bridge-builder will determine the extra weight, beyond the weight likely to come upon a bridge, which it should be calculated to sustain. From the examples given (See abstract, page 49), it appears that the difference of opinion upon this subject is very great; wooden bridges are thought to be safe, if they are limited to carry a weight six and eight times more than it is probable will be placed upon them. Stone bridges, from 15 to 400 times more. Iron bridges, when insistent, 78 times more; but when pendent, an excess of strength for insurance is disregarded, though the hazard is augmented by the motion and jerks* contingent to such bridges.

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* Le bélier hydraulique de Montgolfier may illustrate analogically this motion. See Mollet's Hydraulique Physique, 1809.

EXAMPLES. — Let there be allowed in the calculation for insurance weight upon a foot superficial at the vertex of an arch, fifteen times the real incumbent weight.

Let the real incumbent weight per foot superficial at the vertex of an arch be taken at 250lbs. avoirdupois. Then the real weight + the insurance weight = 250 + (250 × 15) = 4000lbs., a constant quantity being nearly the same as it will be deduced, the bridge of Neuilli is capable of sustaining on a foot superficial at the vertex. In timber bridges, the real incumbent weight may be taken at 150. Then 150 + (150 × 15) = 2400lbs. But in these examples 4000lbs. are taken. (*a*)

MATERIALS.	Weight of one cubic foot of the material in lbs. avoirdupois. $= g$	Height of a prismatic column one foot square base, weighing 4000lbs. avoirdupois. $= \frac{w}{g} = \frac{4000}{g} = w'$	Modulus of fracture $= f'$ or radius of curvature of an arch at the vertex, supporting only its own weight in	Radius of curvature of an arch at the vertex, when the height of the key is one foot, and the incumbent weight 4000lbs. $= c = \frac{f'}{1 + \frac{w'}{n}}$	Height of the key of an arch, when the radius of curvature at the vertex is 50 feet, or the diameter of the circle of curvature is 100 feet and the incumbent weight 4000lbs. $= n = \frac{w'}{\frac{f'}{c} - 1}$	Versed sine or height of a circular arch, when the height of the key is one foot, the semi-pan 28.11 feet, and incumbent weight 4000lbs. $= x = c - \sqrt{c^2 - y^2}$	Semi-span of a circular arch when the height of the key is one foot, and the versed sine or height is 28.11 feet, and the incumbent weight is 4000lbs. $= y = \sqrt{(2c - x)x}$
		FEET.	FEET.	FEET.	FEET.	FEET.	FEET.
Brick	135.5*	29.52†	858‡	28.11§	1.827‖	28.11	28.11
Portland stone. .	151.75	26.36	4337	158.51	.307	2.513¶	90.118 **
Cornish granite .	166.375	24.04	5502	219.72	.220	1.806	107.529
Aberdeen granite	164.06	24.38	9581	377.50	.127	1.049	142.943
Fir	37.5	106.67	7680	71.33	.699	5.774	56.740
Oak	52.81	75.74	10906	142.11	.348	2.808	86.847
Cast iron	439.56	9.1	51229	5072.17	.009	.078	558.996

(*a*) The heights of the keys obtained in the column of the heights of the key of an arch will not be found by a comparison with those in the column of the specific height of the key in the Tables of Bridges in brick to vary more than might have been anticipated. In respect to the other materials, the same conclusion may be drawn, upon the supposition, that arches with such materials should be constructed with ribs or arcs doubleaux, after the practice of the Gothic architects: for example, suppose a bridge were erected of Cornish granite with ribs five feet from the middle of one to the middle of another, and a foot wide, then the height would be 1.1 feet. In cast iron, and the ribs $\frac{3}{4}$ of an inch wide, the height would be 0.72 feet; and the limit of the radius of curvature, which in the preceding column is 5072.17., would become, on the same supposition, only 70.273 feet. In fir, if the incumbent weight be taken at 2400lbs., and the interval between the ribs double the width of them = 2 feet, then the height of the key will be 1.26 feet, and a similar increase would take place in the versed sine, and decrease in the semi-span.

The third column of the table in M. Patte's Edit. Blondel's Cours D'Archi, vol. vi. 1757. page 194. contains the thickness of vaults at the key, which thickness has been determined, after a number of experiments, as he says, by taking the twenty-fourth part of the diameter of a semi-circular arch, to which one foot must be added, and one line per foot taken away from that sum. The same rule will serve for vaults surbaissées, taking the double of the radius of curvature at the vertex for the span of the arch.

Perronet, Desc. des Proj. &c. des Ponts, page 625, says: "It is the custom to make the length of the key for great arches which are 'surbaissées au tiers' the twenty-fourth part of their diameter. It is proper to give to keys of small arches a foot in height more than this twenty-fourth part, and to diminish this sum in the ratio of one line for each foot of the span of the arch, so that the key will be reduced to 27 and to 42 inches for arches of 36 and 72 feet. Semi-circular arches require a key a little shorter:" and again, in page 664, he says, "The thickness of semi-circular arches has been fixed, by a Treatise I have written on the subject, at the twenty-fourth part of their diameter, adding one foot to it, and diminishing the sum in the ratio of one line for every foot of the diameter. For arches surbaissées, the radius of curvature at the vertex being doubled for the diameter, proceed as with semi-circular arches.

* $\frac{2168}{16} = 155.5$lbs. See table of the strength of materials. † $\frac{4000}{135.5} = 29.52.$ ‡ See table. § $c = \frac{f'}{1 + \frac{w'}{n}} = \frac{858}{1 + \frac{29.52}{1}} = \frac{858}{30.52} = 28.11.$

‖ $n = \frac{w'}{\frac{f'}{c} - 1} = \frac{29.52}{\frac{858}{50} - 1} = 1.827.$ ¶ $x = c - \sqrt{c^2 - y^2} = 158.51 - \sqrt{158.51^2 - 28.11^2} = 2.515.$ ** $y = \sqrt{(2c - x)x} = \sqrt{\{(2 \times 153.51) - 28.11\} 28.11.} = 90.118.$

TO DETERMINE THE WEIGHT AN ARCH IS CAPABLE OF SUSTAINING ON A FOOT SUPERFICIAL AT THE VERTEX, WHEN THE RADIUS OF CURVATURE THERE, THE HEIGHT OF THE KEY, AND THE WEIGHT AND STRENGTH OF THE MATERIAL, ARE GIVEN.

STONE BRIDGES.

EXAMPLE.

Bridge of the Holy Trinity, Florence.
(White Statuary Marble, not veined.)

This arch is pointed at the vertex, and the angle of the joint with a vertical line is 5° 56′, the secant of which varies so little from 1, that the arch may be considered to be round headed.

By the table of stone bridges, $c = 172.63$ feet.
$n = 2.75$ feet.

By the table of the strength of materials, $f' = 5058$ feet
$g = \frac{2760}{16} = 172$ lbs.

By the formula $w' = n\ (\frac{f'}{c} - 1) = n\ (\frac{5058}{172.63} - 1)$ $= 2.75 \times 28.3 =$ a prismatick column of the same material 77.8 feet in height of one square foot base, and 77.8×172 lbs. $= 13382$ lbs. on a foot superficial, which is the limit of weight the bridge is capable of sustaining at the vertex.

Bridge of Neuilly.

Perronet, in his Descr. des Proj. et de la Cons. des Ponts, pages 616 and 624, states, that a cubic foot of Saillancourt stone, of which this bridge is constructed, weighs 152 French pounds; that a cubic foot will just bear a column of the same material 1580 feet high; that the radius of curvature at the vertex (originally 150 feet) of the arch, after it had taken its bearing, to be 244 feet, and the height of the key stone to be five feet (French measures) $c = 244$, $n = 5$, $g = 152$ lbs., $f' = 1580$, $w' = n\left(\frac{f'}{c} - 1\right) = n\left(\frac{1580}{244} - 1\right) = 5 \times 5,475 =$ a prismatick column of the same material, one square foot base, 27.4 feet high, and $27.4 \times 152 = 4165$ lbs. French, on a foot superficial, which is the limit of weight the bridge is capable of sustaining at the vertex. To find the weight in English avoirdup. pounds on an English foot, as $12.78^2 : 12^2 :: 4165 : 3672$ French lbs. on an English foot. Then, as $7000 : 7561 :: 3672 : 3966$ English lbs. avoird. on an English foot.

Waterloo Bridge, London.

(Cornish Granite.)

$c = 112.5$ feet, $n = 5$ feet, $g = \frac{2662}{16} = 166.4$ lbs. $f' = 5502$ feet, $w' = n\left(\frac{f'}{c} - 1\right) = n\left(\frac{5502}{112.5} - 1\right) = 5 \times 47.9 =$ a prismatick column of the same material 239.5 feet high of one square foot base, and $239.5 \times 166.4 = 39846$ lbs. avoirdupois on a superficial foot, which is the limit of weight the bridge is capable of sustaining at the vertex.*

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* The same may be determined directly in lbs. as follows: $c = 112.5$ $g = \frac{2662}{16 \times 64} = 2.6$ lbs. $f = 14302$ lbs. $n = 5$ feet, $= \frac{120}{3}$

Dunkeld Bridge, Perthshire.

(Aberdeen granite.)

$c = 48.5$ feet, $n = 3.17$ feet, $g = \frac{2625}{16} = 164$ lbs. $f' = 9581$ feet, $w' = n\left(\frac{f'}{c} - 1\right) = n\left(\frac{9581}{48.5} - 1\right) = 3.17 \times 196.54$ = a prismatick column of the same material of one square foot base 623 feet high, and $623 \times 164 = 102172$ lbs. avoirdupois on a superficial foot, which is the limit of weight the bridge is capable of sustaining at the vertex.

The foregoing bridges are not built with ribs or arcs doubleaux.

WOODEN BRIDGES.

Tête, Picardy.

(Fir)

$c = 319$ feet, $n = 3.834$ feet, $g = \frac{600}{16} = 37.5$ lbs. $f' = 7680$ feet, $w' = n\left(\frac{f'}{c} - 1\right) = n\left(\frac{7680}{319} - 1\right) = 3.834 \times 23.07$ = a prismatick column of the same material, one square foot base 88.45 feet high, and $88.45 \times 37.5 = 3317$ lbs., and $\frac{3317}{3}$[*] $= 1106$ lbs. avoirdupois

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= 40 times 1½ inches, $w = n\left(\frac{f}{c} - g\right) = n\left(\frac{14302}{112.5} - 2.6\right) = 40 \times 124.5 = 4980$ lbs. on a linear foot 1½ inches wide and $4980 \times 8 = 39846$ lbs. as before on a foot superficial.

[*] A wooden bridge being generally composed of frames at a distance from each other, equal to twice the width of the frame,

on a foot superficial, which is the limit of weight the bridge is capable of sustaining at the vertex.

Bamberg, Germany.

(Fir.)

$c = 321.5$ feet, $n = 4.75$ feet, $g = \frac{600}{16} = 37.5$ lbs. $f' = 7680$ feet, $w' = n\left(\frac{f'}{c} - 1\right) = n\left(\frac{7680}{321.5} - 1\right) = 4.75 \times 22.89 =$ a prismatick column of the same material, one square foot base, 108.73 feet high, and $108.75 \times 37.5 = 4077$ lbs., and $\frac{4077^{*}}{3} = 1359$ lbs. avoirdupois on a foot superficial, which is the limit of weight the bridge is capable of sustaining at the vertex.

IRON BRIDGES.

Southwark, London.

(Cast English iron.)

There are eight parallel ribs and two diagonal ones $2\frac{1}{2}$ inches thick, the bridge is 45 feet wide, therefore the weight sustained by each is 4.5 feet.

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the weight 3317.6 must be divided by 3, then $\frac{3317.6}{3} = 1106$ lbs. on a foot superficial. It seems extraordinary that, notwithstanding the examples of stone bridges erected by the architects of the middle ages, notwithstanding the vaultings of ecclesiastical buildings, and the universal practice in respect to wooden and iron bridges, that stone bridges should be erected, to the great waste of material and labour in modern times, of a uniform height of key, from face to face, and not with ribs or *arcs doubleaux*.

$c = 312$ feet, $n = 6$ feet, $g = \frac{7033}{16} = 439.56$ lbs. $f' = 51229$ ft. $w' = n\left(\frac{f'}{c} - 1\right) = n\left(\frac{51229}{312} - 1\right) = 6 \times 163.19$ = a prismatick column of the same material one square foot base 979.14 feet high, and 979.14 × 439.56 lbs. = 430390 lbs. and because the ribs are only $2\frac{1}{2}$ inches thick, and there are in 4.5 feet, 21.6 times $2\frac{1}{2}$ inches, $\frac{430390}{21.6} = 19926$ lbs. avoirdupois on a foot superficial, which is the limit of weight the bridge is capable of sustaining at the vertex.

Menai Bridge,[*] *Bangor, North Wales.*
(Wrought English iron.)

Mr. Telford states that the weight per foot superficial to break down the bridge is $\frac{1674 \times 2240\,\dagger}{566.5 \times 30} =$ 221 lbs. per foot superficial, derived from an experiment made on a bar of good iron, one inch square, by a weight suspended from it in a curve of a catenary precisely that of the Menai bridge; in which the fibres of the material must have lost in strength from the deflection, those on the concave side suffering compression, and on the convex side extension.

At the Runcorn bridge, 60 lbs. per foot superficial $\frac{800 \times 2240}{1000 \times 30} = 60$ was the limit: see page 248 Mr. Barlow's essay; though the strength of iron in the computation was derived from an experiment upon it *vertically* suspended, and 27 tons assumed as the breaking weight, which, in such a curvature as the angle 11°15′ at the point of suspension, gives, should not have been assumed but with very great allowance. (See also Tract 3, page 29.)

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[*] See tract 3, page 37. Dr. Young, lec. 13, Nat. Phil., says, "Although iron is much stronger than wood, yet it is more liable to accidental imperfections; and when it fails, it gives no warning of its approaching fracture."

[†] See page 25, 3d Report, Menai bridge, House of Commons, 29th April, 1819.

Abstract, shewing the limit of weight on a foot superficial at the vertex, the following bridges are capable of sustaining, derived from the foregoing calculations, supposing them equally strong in respect to compression in every other part; also the probable and insurance weights.

	Architect.	Material.	Limit of weight in lbs. avoirdup.	Probable weight in lbs. avoird.	Insurance or excess for contingencies, lbs. avoirdupois.
The bridge of the Holy Trinity, at Florence	Ammanati	Statuary marble, not veined	13313	250	13063
— Neuilli, near Paris	Perronet	Saillancourt stone	3966	250	3716
— Waterloo, London	Rennie	Cornish granite	39846	250	39596
— Dunkeld, Perthshire	Telford	Aberdeen granite	102172	250	101922
— Tête, Picardy	Coffinette	Fir	1106	150	956
— Bamberg, Regnitz, Germany	Weibeking	Fir	1359	150	1209
— Southwark, London	Rennie	Cast-iron *	19926	250	19676
—Menai pendent bridge, Bangor, North Wales	Telford	Wrought-iron	221		
— Proposed pendent bridge at Runcorn	Telford	Ditto	60		

Gauthey says (vol. 1, page 67), "The great thickness which the ancients have every where given to the supports in their edifices, lead to the conclusion that they had little idea of the resistance of material;

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* The excess in respect to the bridges of Bonar and Craigellachie could not be calculated for want of the distance between the ribs and the thickness of them; but by referring to the date of their erection, and the note page 33, it may be presumed that they are 40 times weaker than intended in respect to compression.

When it is understood to what extent the expence of public works is increased in order to insure the risk arising from a want of knowledge; probably there may arise a Colbert to limit the waste, by causing an inquiry to be made into the qualities of matter applicable to building purposes in respect to tension and compression, and by determining measures of fracture, and measures of strength.

on the contrary, the architects of the middle ages convince us that they had studied, in this respect, the properties of stone;" which observation will be confirmed by reflecting upon the construction of their buildings; they substituted buttresses for walls of uniform thickness, ribs for arches of uniform thickness, granite for ordinary stone in their slender columns, and toph for pannels. Their vaults resemble the backs of the skeletons of quadrupeds, and their buttresses varied in their directions almost with the correctness that the directions of the legs of animals vary according to the direction of the pressure they receive. Ministers of the Deity, they studied his laws, and humbly corrected or abandoned their erroneous hypotheses, by a reference to the successful practice of mankind, and by the analogous examples which pervade His works.

GEOMETRICAL PROBLEMS TO DETERMINE THE RATIO OF THE PARTS OF POSTS, PIERS, AND WALLS, AND OF BEAMS, LINTELS, AND ARCHES.

TO DESCRIBE A LOGARITHMIC CURVE TO A GIVEN SUBTANGENT.

Fig. 1. Let the horizontal line CV be equal to the subtangent, and CH at right angles to it be the assimptote or axis of the curve. Make the angle VCR = 56°.28′, and the angle VCE = 33°.32′ its complement, and draw through V, VER parallel to the axis, cutting CE and CR in E and R.* With C as a centre, and radius CE, cut the axis in L, and draw LG parallel to CV. With R as a centre, and radius RV, cut RC in B, and make LG equal to CB. LG is an ordinate and a mean proportional to CV one extreme. Through Y at a distance YL equal to CL, draw AW parallel to CV. From Y set off YA equal to CV and on the axis, YD equal to LG. With the centre in YA, describe a semicircle through D and A, cutting AW in W. YW is another ordinate, and in like manner find the ordinate HK, and through KWGV

draw the logarithmic curve, whose subtangent at every point equals CV.

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* Or draw V*e* parallel to the axis, and with C as a centre and radius C*e* = CV + $\frac{CV}{5}$, cut V*e* in *e*, join C*e*, and draw CR at right angles to it, cutting *e*V continued in R.

CV may be used as a scale in the subsequent operations, this operation will be nearly correct, the co-secant of 56° 28′ being nearly 1.2 to 1 radius.

To find an intermediate ordinate SO,

Bisect the distance CL in S, and through S draw SO parallel to CV. Make ST in the axis on one side S equal CV and SX in the axis on the other, equal LG and on TX as a diameter describe the semicircle TOX, cutting SO in O, SO is the mean to CV and LG extreme proportionals. In like manner, by extreme and mean proportionals, any point in the curve may be found.

Put $YW = n$, $YL = h$, $CV = f'$, $m = 2.302585$, $LG = w$.

Then $\text{Log. } w = \text{Log. } n + \frac{h}{f'm}$

$h = f' m \, (\text{Log. } w - \text{Log. } n.)$

TO DESCRIBE THE CATENARY EQUALLY STRONG IN EVERY POINT, AND TO DETERMINE ITS INTRADOS.

Let the vertical line CA be the radius of curvature at the vertex. Through A draw AR at right angles to CA, and with CA radius and C centre, draw the quadrant of a circle AB, and the right line CR, making any angle with CA, cutting the quadrant AB in m and AR in R. Continue the ordinate DE, taken equal to CA, and drawn to a logarithmic curve, whose axis or assimptote is in the line AR, and whose subtangent equals CA, and upon it make DL equal CR. Fig. 2.

Draw LM parallel to the assimptote of the logarithmic curve, and cutting that curve in M. From A, the vertex of the arch, take the absciss AI equal to LM, and draw the ordinate IH equal in length to the circular arc Am. In the same manner find any other point U. Through AHU, draw the catenary required, whose ordinate can never exceed AB or the length of an arc of 90 degrees of its osculatory circle at the vertex. From the point H draw HN parallel and equal to RC, HN is the radius of curvature to the point H, which is always equal to c Sec φ putting c for the radius of curvature CA at the vertex, and φ for the angle AHI, the curve makes with its ordinate, and in like manner find another radius of curvature ZG, through CGN draw the involute of the curve AH.

At any point H draw HP parallel and equal to AV, the height of the key of the arch at the vertex, and draw the horizontal line PF, cutting the radius of curvature HN in F. In like manner find any other point O. Through VFO draw the intrados of the arch required, which is a curve having a maximum ordinate, which takes place when its absciss $= cm$ Log $\sqrt{\frac{c}{n}}$, when n represents the thickness VA at the vertex, c the radius of curvature CA, and $m = 2.302585$.*

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* This catenary AHU may also be supposed the intrados of an arch, in which case the extrados becomes a curve of contrary flexure, according to the dotted line, which takes place when its absciss $= cm$ Log. $\sqrt{\frac{c + 2n}{n}}$, or it may be supposed any line between the extrados and intrados, in which case the extrados will be a curve of contrary flexure, and the intrados a curve having a maximum ordinate. The key of an arch may be supposed to be the aggregate of the keys of numerous arches riding one upon another, each increasing in thickness from its vertex as the secant of the angle, its extrados, intrados, or some intermediate line makes with its ordinate, in which case the joints of each subjected arch may approximate very nearly to the mathematical desideratum, "that a due curve should pass through the centre of gravity of, and at right angles to, each lamina considered as a voussoir;" but as this vision can never be realised, it is the architect's duty to assume the thickness to be at right angles to the

To draw a line at right angles to the tangent to the intrados VFO at any point *a*.

From the point *a*, draw the vertical line *ag* equal to CA, the radius of curvature at the vertex and the horizontal line *gi*. Continue *ab*, the thickness of the arch, at right angles to the extrados at *b*, cutting *gi* in *i*. Take in *ag*, *ac* equal VA, and draw the horizontal line *cb*, cutting *ai* in *b*. With the radius *ab* and centre *a*, cut *ag* in *d*, and draw the horizontal line *de*, cutting *ia* in *e*. Draw the vertical line *ik* equal to *ae*. Join *ka*, which is the line required. Fig. 2 and 3.

TO FIND THE INTRADOS WHEN THIS CATENARY BECOMES A STRAIGHT LINE, OR THE ARCUATION AND FLEXIBILITY CEASES; THE LINES TS, DE, DRAWN AT RIGHT ANGLES TO THE EXTRADOS, BEING PARALLEL.

When a horizontal line. Let AD be the horizontal line. With the centre A, and any radius AI in the vertical line ACO, cut AD in T. Draw TS at right angles to AD and IH horizontal, cutting the extrados of the arch at the point H. Make TS equal HF, the thickness of this catenary at right angles to it at H. In like manner find any other points E, and Q. Through VSEQ draw the curve VSEQ, the intrados required, which is a logarithmic curve whose subtangent equals the radius of curvature of the catenary AHU at the vertex A, from which it is derived. The logarithmic curve may be drawn above or below the horizontal line AD. Fig. 2.

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extrados, and the joints at right angles to the intrados, which hypothesis is happily favourable to the strength of the material, to the stability of the arch, and conformable to the necessities of practice.

In both cases, the extrados with its intrados will represent a beam*, bressummer, lintel, or girder, whose fibres or laminæ are horizontal, equally strong in every part; in the first case, a weight would affect the rigidity of the material; in the second case, the cohesiveness of the material.

When a vertical line.

From any point U in this catenary draw the vertical line UK cutting AR in K, and through any point H above U in the catenary draw the horizontal line YW cutting UK in Y. Make YW equal to HF, the thickness of the catenary at right angles to it at H. In like manner, find other points O, and X. Through XWO draw the curve which is a logarithmic curve, whose subtangent equals the radius of curvature of the catenary at its vertex A, from which it is derived, and KYUOWX is a section of a wall equally strong in every part.

TO DETERMINE THE BATTERING OF A POST, PIER, OR WALL, SO THAT ANY HORIZONTAL SECTION OF IT MAY BE PROPORTIONAL TO THE VERTICAL COMPRESSION AT THAT SECTION, AND TO DETERMINE THE WEIGHT IT WILL SUSTAIN.

In the case of a Pier.

Fig. 4. Let CV be the height of a prismatick pier or cylinder, which by its own weight would begin to crush at

its base C, or the modulus of fracture of a given material in respect to compression.

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* Through the cusp O where the intrados VFO of the arch cuts the absciss, draw the horizontal line OQ, cutting the logarithmic curve VSEQ in Q. It happens in this case that VO is to OQ as 4 to 5, and that the logarithmic curve VSEQ is, or a very close approximation to, the semi-curve of the arch of equilibration, which Dr. Hutton, vol. i., Tracts, page 62., terms, "an arch of equilibration, whose extrados shall be a horizontal line."

Let the vertical line ACV be the axis or assimptote of a logarithmic curve DB, FE, drawn on each side of the axis, whose subtangent AC equals 2 CV equals twice the modulus of fracture of the given material. The outlines or curves BD, EF, will then be the batterings of a pyramidal pier, whose base is any regular polygon or circle, (see Figures A, B, C,) its sides infinitely approaching the axis upwards, and infinitely receding from it downwards. The weight GBEH, it will sustain, will be equal to a prismatick pier or cylinder, as the case may be, equal in height to half the subtangent, or $=$ CV the modulus of fracture of a material homogeneous with that of the pyramidal pier, and of an equal area as that of the top BE of any frustrum of the logarithmic solid.

In the case of a Wall, see Plan D.

The logarithmic curves or batterings BD, EF, have a subtangent AC equal to CV equal to the modulus of fracture of the material, and the height of the weight is the same as in the case of a pier. Fig. 5.

If such a pier or wall be turned upside down, and suspended at the thick end in the air, it will be no sooner pulled asunder in one part than another by the weight ascertained as before.

Put CE $= n$, CL $= h$, CA $= f'$, $m = 2.302585$, LM $= w$. Then $\text{Log. } w = \text{Log. } n + \frac{h}{f'm}$

$h = f'm\,(\text{Log. } w - \text{Log. } n)$. If LC be bisected in N, the ordinate NO $= \sqrt{\text{LM} \times \text{CE}}$.

To determine the depth at every part of a beam, bressummer, girder, or lintel, whose vertical sides are Fig. 6.

parallel, and upper or underside horizontal, whose bearing or distance between the supports is AD and depth in the middle BC. Let the horizontal line AC equal $\frac{1}{2}$ AD be the axis or assimptote of the logarithmic curve BE, whose subtangent is equal to the modulus of fracture of the given material. Draw FG at any point F in the horizontal line AC at right angles to AC, cutting the curve in G. FG is the depth required at F. In like manner the depth of any part of the other half of the beam, &c. may be obtained.*

$BC = n$. Subtangent $= f'$

$$\text{Log. } FG = \text{Log. } n + \frac{CF}{f' \times 2.302585}$$

$$FC = f' \times 2.302585\ (\text{Log. } FG - \text{Log. } n.)$$

TO DETERMINE THE SECTION OF AN ABUTMENT WALL, UPON THE SUPPOSITION THAT THE EXTREME POINT OF ITS BASE IS FIXED AND IMMOVABLE.

Considered as a strutt.

Fig. 7. Let GD be the vertical height of an abutment wall, AD a horizontal line where it leaves the foundation or base line, EF the base line of a weight W, and EFG the angle, EF makes with a vertical line.

Draw the horizontal line GEH. In GD continued take DT equal to FG. Upon GT as a diameter, describe the circle GTL cutting AD in L. Make DK in AD equal to 2 DL and HG equal to DL. Join HF and HK. K is a point in a parabola whose focal

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Fig. 6. * By the approximation of principles, a prismatick beam may be assumed to contain within it an arch pendant or insistent, whose height at the key $= n$, versed sine $= x$, and semispan $= y =$ half the distance between its supports, wherein the excess, that is the segment EBH above or below the arch, as the case may be, must be considered part of the weight. See Table of Examples, two last columns.

distance is FG. HKD is the angle of thrust due to the height of the pier = HFG the angle due to the cousinet HEF into which the angles of any weights WHG, WHF, WEF, WGF, whose bases do not exceed HF, nor make a greater angle with a vertical line than the angle HFG, resolve. Such weights increasing in length as the secant of the angle PEN, they make with a vertical line to EP radius = the modulus of fracture of the material, while they diminish in width EF as the secant of the angle GFE to GF radius = the modulus of horizontal thrust.

Draw KM parallel to HF and TM parallel to AD intersecting in M.

FEHF is the section of the *cousinet* or base of the weight.

HKMTFH the section of the abutment wall.

If the abutment wall be required to be equally strong in every part.

Let the vertical line KT be the axis of a logarithmic curve IHA, whose subtangent equals the modulus of fracture of the material. Fig. 8.

Draw HK making an angle KHG with the axis equal to the angle HFG of thrust before obtained, and touching the logarithmic curve. From K take KG in the axis equal to the subtangent, and draw GH horizontal, cutting the curve in H, the point of contact. From G set off GD on the axis equal the height of the pier, and DT equal to GF the modulus of horizontal thrust and draw DA cutting the curve in A. Draw the tangent AL to the point A, and AB at right angles to AL, cutting TB drawn parallel to AD in B. Draw HF at right angles to HK cutting the axis in F, then HFTBAH is the section of the abutment wall required.

The direction of the weight considered to be the tangent of the extrados of an arch.

The direction EW is resolvable into the extrados of any arch EA, the tangent of which is at right angles to any line EF drawn from F to E any point in HG whose radius of curvature at the vertex A does not exceed the modulus of fracture of the material when there is no incumbent weight, and when there is, does not exceed $c = \frac{f'}{1 + \frac{w'}{n}}$, see page 43. of this tract, putting $GF = n = AV$, and when the abutting joint EF does not exceed HF.

Let the extrados EA be a circular arc, and the vertical line CA the radius = EFC. From any point *a* in the extrados draw *ac* at right angles to the tangent at *a*, and let fall from *a* the vertical line *ab* equal GF, and draw the horizontal line *bc*, cutting *ac* in *c*. Through V*c*F draw the intrados to the extrados A*a*E.

An abutment wall considered as an arch.

Fig. 9. Take GF = the modulus of horizontal thrust = the focal distance of a parabola in GD the height of the abutment wall = the absciss GD of the parabola GBK. Draw the horizontal line GH of any length, and bisect it in E and join EF. From EH let fall the similar triangle EHB, EH and GF homologous sides, B is a point in the parabola: in like manner, find any other point K. Through GBK draw the parabola GBK, cutting the horizontal line DK in K. Continue WE at right angles to EF the direction of a weight whose base is equal to EF till it touch the parabola.

Find T, M, K, as before.

From B, the point of contact, draw BN equal and parallel to EF, N is a point in the intrados.

Let BN be in the radius of curvature of any curve, B*a*A the extrados of an arch, BW being the tangent at B, and from any point *a* in the extrados KB *a* A, let fall the vertical line *ab*, and draw *bc* horizontal, cutting the radius of curvature to the point *a* in *c*. Through V *c* NM draw the intrados.

It is manifest that if any semi-lunular VIT MNFV be supposed, and a component part, the arch will be strengthened thereby.

An abutment wall resisting an uniform horizontal force, such as that of the wind.

If the opposing side GD be a vertical line, the Fig. 10. other side will be a parabola GK; the modulus of horizontal thrust at any part equals GF the focal distance, the parabola GK limits the range of any parabola whose vertex is any point *g* in GD and focal distance equal GF.

An abutment wall supporting a fluid or bank of earth.

If the opposing side GD be a vertical line, the Fig. 11. other side will be an inclined plane, the angle of inclination varying as the slope, the fluid or earth naturally assumes from the limit a vertical line, at which stone will naturally stand, to the limit a horizontal line which a perfect fluid naturally assumes.

An inclined plane limits the parabolas which will pass through GD at a given angle.

When the pressure is from a fluid, the vertices of the parabolas are in GD, and the angle of inclination of the wall or of thrust GKD will be 45°.

If the natural slope of the earth K′E be at an angle EK′D, then the slope of the wall will be K′G bisecting the angle IK′E which EK′D wants of 90°.

TO DETERMINE THE EXTRADOS OF AN ARCH AND OF ITS PIERS.

Fig. 12. Let CV = c = the radius of curvature at the vertex of the extrados of an arch.

GF = gf = LY = ON = AV = n, the height of the key. AE = VM, or Ae = Vm = x = the versed sine of the intrados at any point F, f, BEF a horizontal line at the springing of the arch. KLOD a horizontal and base line, below which the foundation is incompressible in every direction. VD or vd = m = the height from the vertex of the extrados, to any ordinate KD or kd to it, OF = VD — VE = VD — $\overline{\text{EA} + \text{VA}}$ = $m - \overline{x + n}$, the height of the pier to the springing of the intrados from the base line.

CONSTRUCTION.

In OF continued each way, take FG and ON each equal AV. On the diameter NG describe the circle, cutting KLOXD in L and X. Make OK equal LX. Draw LY parallel and equal to ON. Join KY and YN. At right angles to KY is the direction of the lateral pressure, and at right angles to YN of the vertical pressure, or pressure in the direction of gravity.

From F at any angle, GFH less than the angle KYL, draw FH, cutting the horizontal line GI in H. Make HI equal GH, and let fall the triangle HIK′ similar to the triangle FGH, HI, and FG homologous sides. Through KK′G draw the curve, which is a parabola. Fig. 12.

Through any point *f* in the intrados of the arch, draw *fg* parallel and equal to AV, and *gi* horizontal, cutting *fh* drawn at right angles to the tangent at *f* in *h*; make *ih* equal *gh*, and from *hi* let fall the triangle *hik* similar to *fgh*, *fg* and *hi* homologous sides. Through V*k* draw the extrados to the arch, which will touch the parabola GK′K in some point K′. From F at right angles, to the tangent at F, draw FK″, cutting the extrados in K″. FK is the abutting joint between the arch and pier. *vk* K′K is the extrados of the arch, and its abutment pier, and is a curve, limiting the greatest range of any body projected from V*g*G at the height VA, above any point A, *f*, F, O, of the intrados of the arch and pier, with the horizontal velocity VA.

From any point K in the extrados, let fall KW parallel and equal to VA, and draw the horizontal line WY, cutting KY, drawn at right angles to the tangent at the point K in Y. KY is the secant of the angle YKW to AV radius. Through YY′ *y* draw the curve A *y* Y′Y. Then the arch and its abutment always contains in its thickness an arch VA*y*Y′YKK′*k*V, whose thickness, in a direction at right angles to the extrados = AV × secant of the angle the extrados makes with its ordinate, and whose strength in every part is proportional to the compression on that part.

CALCULATION — ARCH AND ABUTMENT PIER.

Fig. 12. $OG = m - x$ and put any angle $ykw = gfh = \varphi$.

As $ON : OL :: OL : OG$

$n : OL :: OL : m - x$

$OL^2 = nm - nx$

$OL = \sqrt{nm - nx}$

$DK = EF + 2\sqrt{nm - nx}$

As $KW : WY ::$ rad. : tang. φ

Tang. $\varphi = \frac{WY}{KW} = \frac{WY}{n}$ $\therefore$ $WY = OL = n$ tang. φ

As $KW : KY ::$ rad. : sec. φ

Sec. $\varphi = \frac{KY}{KW} = \frac{KY}{n}$

$\therefore$ $KY = n$ sec. φ

$DK = EF + 2\,n$ tang. φ

As $fg : gh :: hi\ (= gh) : ik = Vd - Vm$

$n : gh :: gh : m - x$

$gh^2 = nm - nx$

$gh = \sqrt{nm - nx}$

$dk = ef + 2\sqrt{nm - nx}$

As $fg : gh ::$ rad. : tang. φ

Tang. $\varphi = \frac{gh}{fg} = \frac{gh}{n}$

$\therefore$ $gh = n$ tang. φ

As $fg : fh ::$ rad. : sec. φ

Sec. $\varphi = \frac{fh}{fg} = \frac{fh}{n}$

$\therefore fh = n$ sec. φ

$dk = ef + 2\,n$ tang. φ

$ky = fh$

TO DETERMINE THE HORIZONTAL THICKNESS OF A BEARING WALL OF A BRIDGE.

(*Commonly called a Bearing Pier.*)

Make BB′* in the springing line FEB′ of the arch, continued equal to the semiparameter of the parabola GK′K equal 2 AV, then BB′ is the width of the bearing pier at that level. Fig. 12.

Bisect BB′ in J. J is a point in the axis of the bearing pier, and in the axis of a logarithmic curve, whose subtangent equals CV, the radius of curvature of the arch at the vertex of the extrados V, and JB is an ordinate to that curve, from which US may be obtained, and any other ordinate RT. If UJ be bisected in R, then RT is a mean proportional to BJ and SU. And by bisecting RU in *r*, the mean proportional *rt* to TR and SU will be had.

Take UZ in UJ, continued equal to CV, and draw SZ, which is the tangent to BS at S; in like manner, by setting up the height CV from any point R in the axis, the tangent to the corresponding point T in the ordinate will be obtained.

CALCULATION.

Put $\text{BJ} = \text{AV} = n \qquad \text{CV} = c$

$\text{Log. US} = \log. n + \frac{\text{JU}}{\text{C} \times 2.302585}$

$\text{JU} = c \times 2{,}302585\ (\log. \text{US} - \log. n.)$

When R bisects JU, then $\text{RT} = \sqrt{\text{BJ} \times \text{US}}$

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* Perronet says in his Mémoire sur l'Epaisseur des Piles, page 620. « Les meilleurs constructeurs, et ceux qui ont fait les plus grands ponts, tant en France qu'ailleurs, n'ont point donné, à

TO DETERMINE THE CURVE, LIMITING THE EXTREME POINTS OF RANGE OF BODIES, PROJECTED WITH A GIVEN HORIZONTAL VELOCITY IN A HORIZONTAL DIRECTION AT HEIGHTS, EACH EQUAL TO THE HORIZONTAL VELOCITY, ABOVE ANY POINT OF A CURVE; OR THE EXTRADOS TO A GIVEN INTRADOS OF AN ARCH AND OF ITS ABUTMENT WALL, ASSUMING THE HEIGHT OF THE KEY-STONE EQUAL TO THE HORIZONTAL VELOCITY.

GENERAL CONSTRUCTION.

Figs. 13. 14. 15. Let A be the vertex of the intrados of an arch, and V the vertex of the extrados. Through any point F in the intrados draw FH at right angles to the tangent at F. Draw the vertical line FG equal to AV, and the horizontal line GI, cutting FH in H. Make HI equal to GH. From HI let fall the triangle HIK, similar to the triangle FGH, FG and HI, homologous sides. K is a point in the extrados.

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beaucoup près, aux piles la même épaisseur qu'aux culées." Page 625. he says, " On a cru devoir leur donner au moins pour épaisseur le double de cette longueur de coupe," speaking of the section of the key or height of it. — Gauthey, vol. 1. page 348. says, of bearing piers, " Quand leur objet est seulement de porter les poids des arches, la résistance de la pierre dont elles sont construites est la principale considération à laquelle on doive avoir égard, et la seule qu'on puisse soumettre au calcul." See the formulæ to determine the height of the key of an arch, page 43.

FORMULÆ TO DETERMINE THE NUMERICAL QUANTITIES IN THE CASES OF THE CIRCLE ELLIPSE AND CYCLOID.

Circle. Put the radius of the circle CF $= r = 50$ Fig. 13. the height of the key AV $=$ GF $=$ LY $= n = 5$, the absciss to the extrados VD $= m = 70$.

Then $v^3 - n\,(m + n - r)\,v = n^2 r$; and where m, n, and r are given quantities, v may be found by the solution of a cubic equation.* $v = 14.5273$.

$$\text{KD} = \left(2 + \frac{r}{v}\right)\sqrt{v^2 - n^2} = 74.2248.$$

$$\text{GH} = \text{HI} = \sqrt{v^2 - n^2} = 13.6385.$$

Also put the angle the intrados makes with its ordinate EF, which is equal to the angle the extrados makes with its ordinate DK $= \varphi$. Then Tang. $\angle\ \varphi = \frac{u}{n} = 2.7277$, which found in a table Nat. Tang. gives the angle $\varphi = 69° \, 52'$. Or secant $\angle\ \varphi = \frac{v}{n} = 2.90546$, which found in a table of Nat. secants, gives the angle $\varphi =$ AFE $=$ GFH $=$ VKL $=$ LYK $= 69°.52'$ as before.

GH $=$ HI $= n$ Tang. $\varphi = 13.6385$, and HF $=$ KY $= n$ sec. $\varphi = 14.5273$ as before.

When CD $= 0$; that is, when VD $= r + n$, the cubic equation becomes $v^3 - 2n^2 v = n^2 r$; from which $v = 12.31$ and KD $= \left(2 + \frac{r}{v}\right)\sqrt{v^2 - n^2} = 68.197$. The sec. $\varphi = \frac{v}{n} = 2.462$, which gives the angle $66°.2'$. GH $= n$ Tang. $\varphi = 11.2479$, HF $= n$ sec. $\varphi = \sqrt{u^2 + n^2} = 12.31$. Put the radius of curvature, at any point of the extrados VK, = R. Then $R = AC + \frac{2VA \times AC^3}{CE^3} = r + \frac{2n}{\cos.^3 \varphi} = r + 2n \sec.^3 \varphi$. At the vertex $R = AC + 2VA = r + 2n$. When $C = 0$, then R is infinite. The evolute has no dependence on the magnitude of the radius AC.

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* See Barlow's New Math. Tables.

Fig. 14. Ellipse.

Put the semi-transverse $CB = a = 50$.
the semi-conjugate $AC = b = 25$.
the height of the key $AV = FG = LY = n = 5$.
the absciss to the extrados $VD = m = 70$.

$$GH = HI = u \text{ and } \sqrt{\frac{a^2 u^2}{b^2 n^2} + 1} = v.$$

Then $v^3 - \left\{ 1 + \frac{a^2 (m-b)}{b^2 \, n} \right\} v = \frac{a^2}{bn}$ a cubic equation from which v may be found when a, b, m, and n are given, in this example falling under the irreducible case. See Barlow's New Math. Tables.

$v = 6.3369$. $GH = HI = u = \frac{bn}{a} \sqrt{v^2 - 1} = \sqrt{mn - bn + \frac{bn}{v}} = 15.65$, $DK = (\frac{2bn}{a} + \frac{a}{v}) \sqrt{v^2 - 1} = 2u + \frac{a^2 u}{bnv} = 80.661$.

Also put the angle the intrados makes, with its ordinate FE, which is equal the angle the extrados makes with its ordinate $DK = \varphi$.

Then Tang. $\angle \varphi = \frac{b}{a} \sqrt{v^2 - 1} = 3.12875$, which found in a table of Nat. Tang. gives the angle $AFE = GFH = VKL = LYK = 72°.17'$.

$GH = HI = n$ Tang. $\varphi = 15.65$.

$HF = n$ secant. $\varphi = \sqrt{u^2 + n^2} = 16.43$.

Putting the two axes equal; that is, when the ellipse becomes a circle, the cubic equation becomes $v^3 - (1 + \frac{m-r}{n})\, v = \frac{r}{n}$ and DK $= (2n + \frac{r}{v}) \sqrt{v^2 - 1}$: whence we have other formulæ for finding v and DK in a circle, besides those before given.

Cycloid. Fig. 15.

Put the radius of the generating circle $= r = 15.9154$.

the circumference of a circle whose diameter is $1 = p = 3.1416$.

the semi-span of the arch CB = the arc ANC $= a = 50$.

the absciss to the extrados VD $= m = 70$.

the height of the key AV = GF = LY $= n = 5$.

the angle the intrados makes with its ordinate MF, which is equal to the angle the extrados makes with its ordinate DK $= \varphi$.

The arc ANE of the generating circle, where the ordinate to the intrados cuts it = A.

The versed of the arc A to radius unity $= v$.

Then $v = \frac{2r+m+n}{2r} \pm \frac{1}{r} \sqrt{(\frac{2r+m+n}{2})^2 - 2mr}$

$$\text{or } v = \frac{2a+p\,(m+n)}{2a} \pm \frac{p}{a} \sqrt{\left\{\frac{2a+p\,(m+n)}{2p}\right\}^2 - \frac{2am}{p}}$$

Either equation being solved, with the values given above, we have $v = 1.78534$; from which the versed sine of the supplemental arc EC $= 2 - 1.78534 = .21466$, and by a table of versed sines, we find the arc EC $= 38°.15'$: and consequently A $= 141°.45$ = and A° $= 141.75°$.

MF $= r$ (sin. A + A° × .017453)

or $\frac{a}{p}$ (sin. A + A° × .017453) $= 49.227$.

$GH = HI = u = \sqrt{mn - \frac{anv}{p}} = 14.42.$

$DK = MF + 2\,GH = 78.067.$

Tang. $\angle\ \phi = \frac{u}{n} = 2.884$, which found in a table of Nat. Tang. gives the angle $AFM = GFH = VKL = LYK = 70^\circ.53.$

$GH = HI = n$ Tang. $\phi = 14.42.$

$HF = n$ sec. $\phi = \sqrt{u^2+n^2} = 15.2675.$

FORMULÆ TO DETERMINE THE RADIUS OF CURVATURE.

			At any point.	At the vertex.
Ellipse	Ordinate	$= y$	$\frac{(c^4+4.\overline{t^2-c^2}\,y^2)^{\frac{3}{2}}}{2tc^4}$	$\frac{c^2}{2t}$
	Transverse axis	$= t$		
	Conjugate	$= c$	$\frac{(t^2p^2+4t.\overline{t-p}\,y^2)^{\frac{3}{2}}}{2t^3p^2}$	$\frac{p}{2}$
	Parameter	$= p$		
Hyperbola	Ordinate	$= y$	$\frac{(c^4+4.\overline{t^2+c^2}\,y^2)^{\frac{3}{2}}}{2tc^4}$	$\frac{c^2}{2t}$ *
	Transverse axis	$= t$		
	Conjugate	$= c$	$\frac{(t^2p^2+4t.\overline{t+p}\,y^2)^{\frac{3}{2}}}{2t^3p^2}$	$\frac{p}{2}$
	Parameter	$= p$		
Parabola	Ordinate	$= y$	$\frac{\overline{p^2+4y^2}\,\rvert^{\frac{3}{2}}}{2p}$	$\frac{p}{2}$
	Parameter	$= p$	$\frac{y(y^2+4x)^{\frac{3}{2}}}{2\sqrt{x}}$	$\frac{y^2}{2x}$
	Absciss	$= x$		
Cycloid	Diameter of the generating circle	$= d$	$2\sqrt{d^2-dx}$	$2d$
	Absciss at any point	$= x$		
Catenary	Constant quantity	$= c$		
	Absciss	$= x$	$\frac{\overline{c+x}\,\rvert^2}{c}$	
	Ordinate	$= y$		c
	Angle the curve makes with its ordinate	$= \varphi$	$\frac{\overline{c\ sec.\ \phi}\,\rvert^2}{c}$	
Catenary of equal strength at every point.	Constant quantity	$= c$		
	Angle the curve makes with its ordinate	φ	$c\ sec\ \phi$	c

Tract 3, page 169.

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* In the equations $\frac{c^2}{2t}$ and $\frac{y^2}{2x}$ exhibiting the radii of curvature at the vertex in the three conic sections, putting radius of curvature equal r, then in the ellipse hyperbola r is a third proportional

to $2t$ and c, and in the parabola to $2x$ and y; the geometrical mode of finding any one of the three quantities, the other two being given. See Tract 3, page 78.

The geometrical methods of finding the radius of curvature to any point in a curve will be found in Maclaurin's Algebra Appendix de linearum geometricarum proprietatibus generalibus.

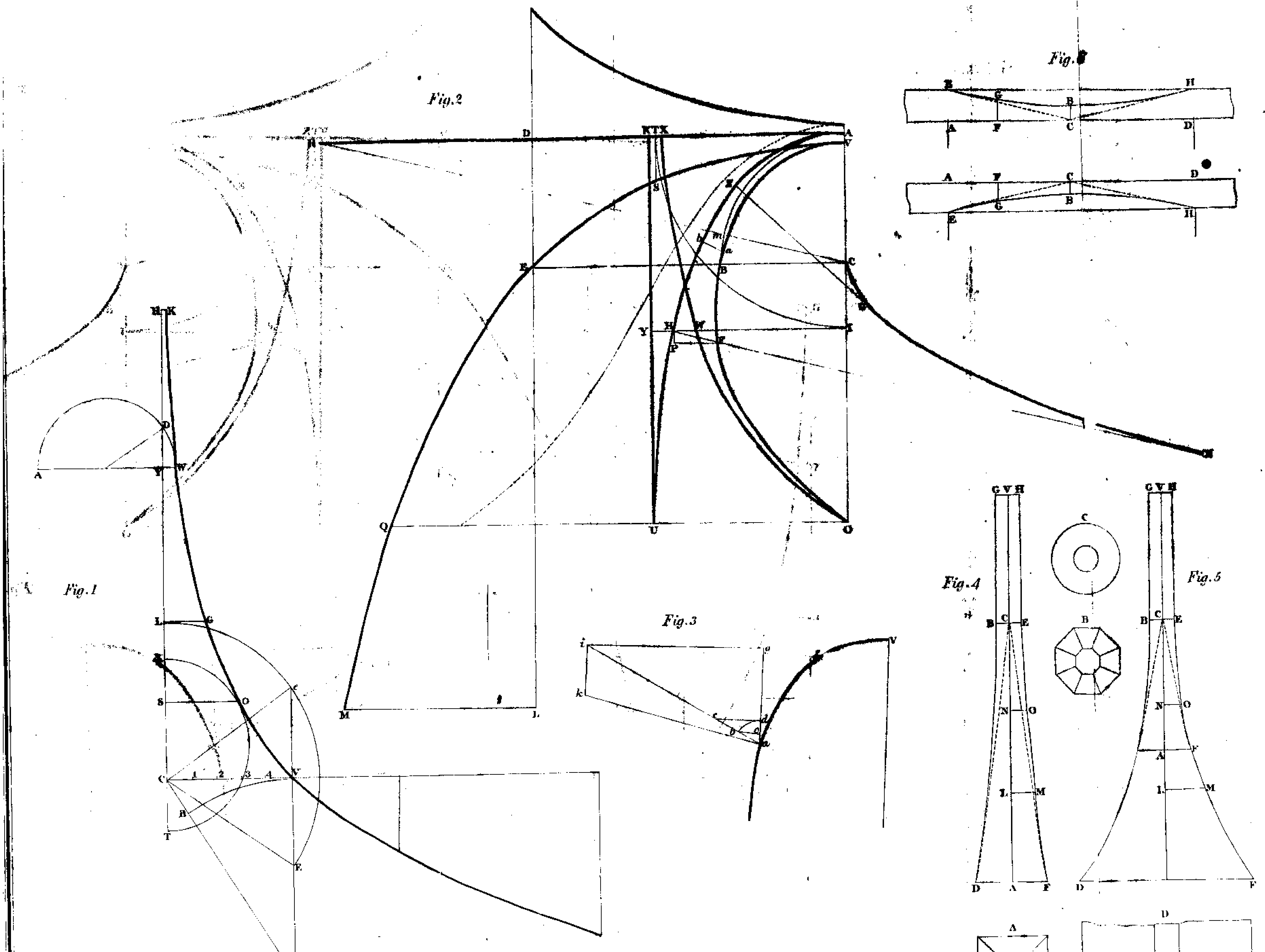
Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

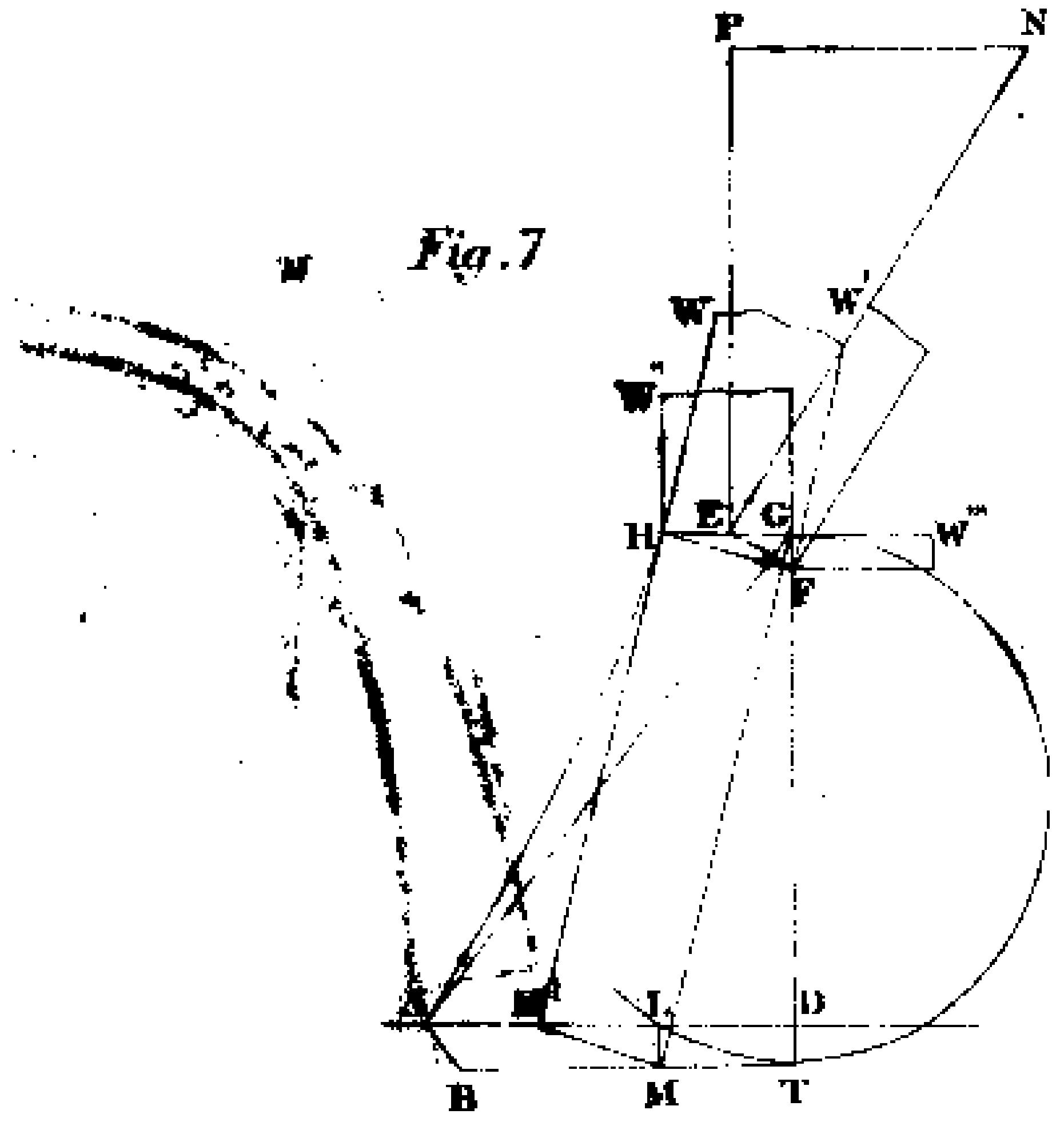

Fig. 8

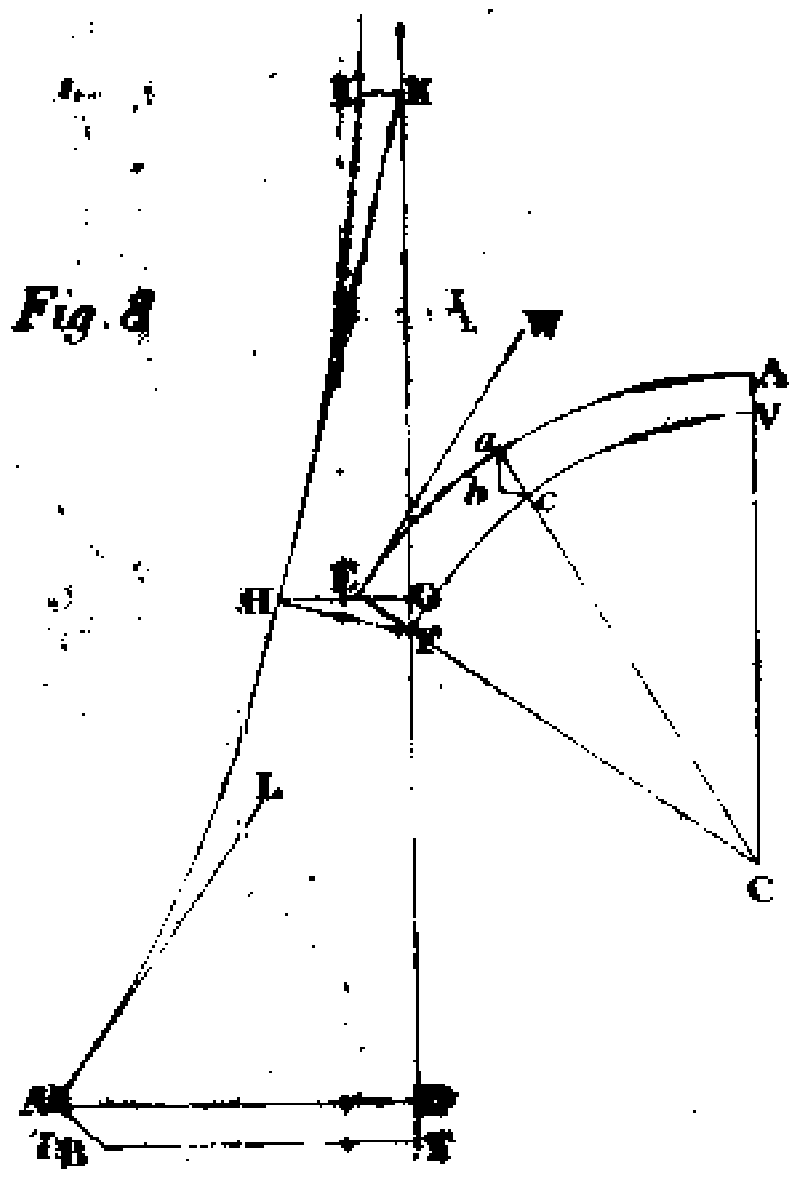

Fig. 9

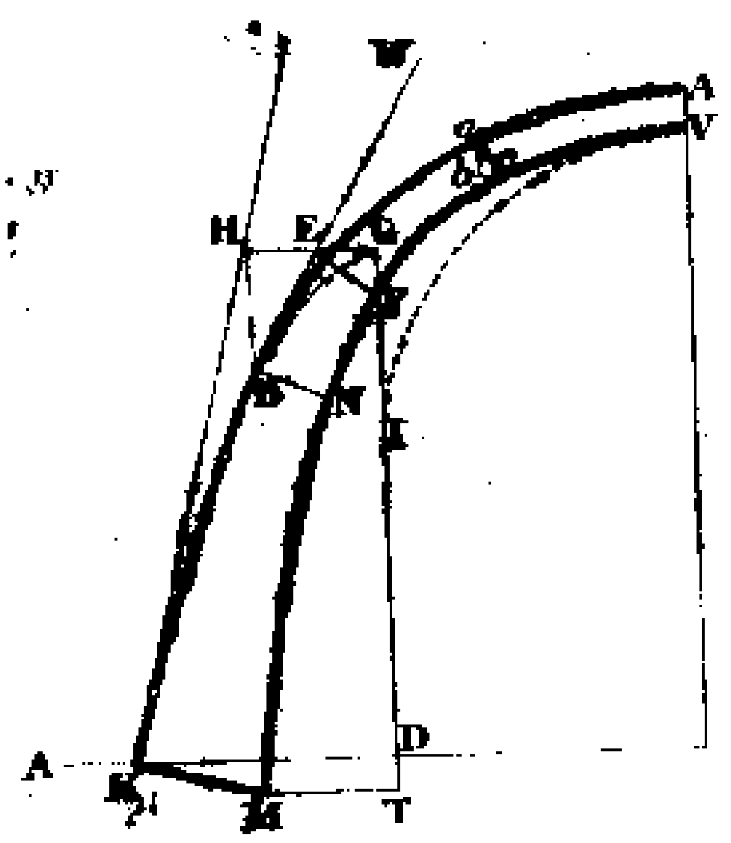

Fig. 12

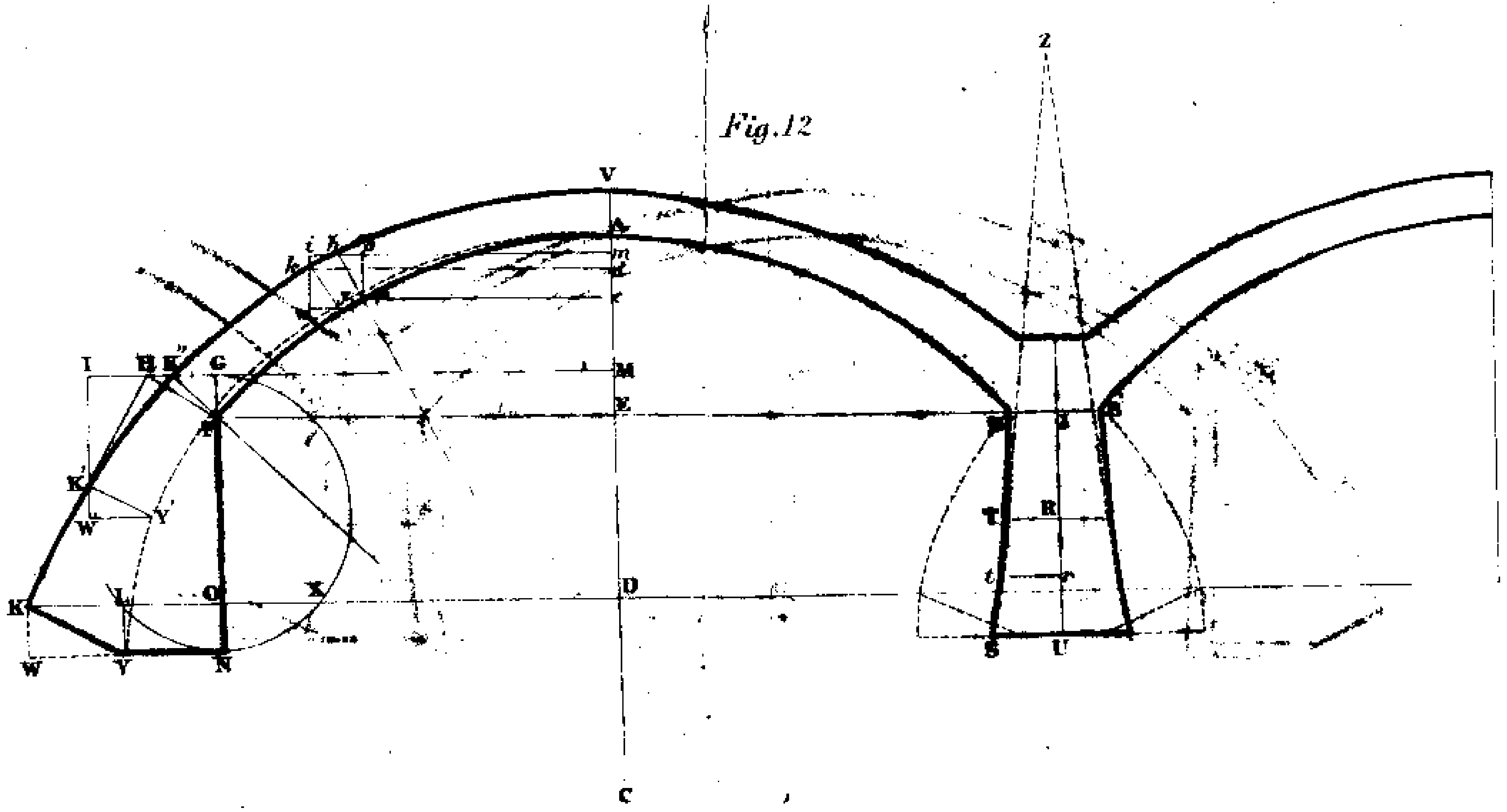

Fig. 10

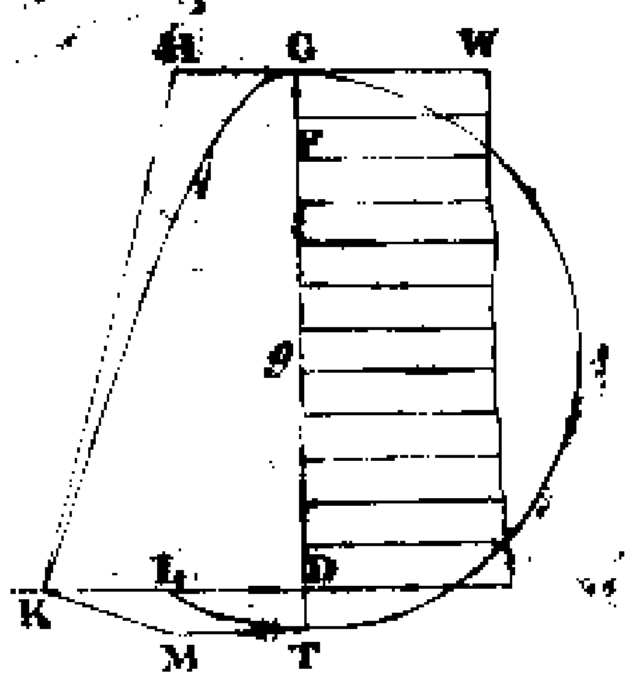

Fig. 11

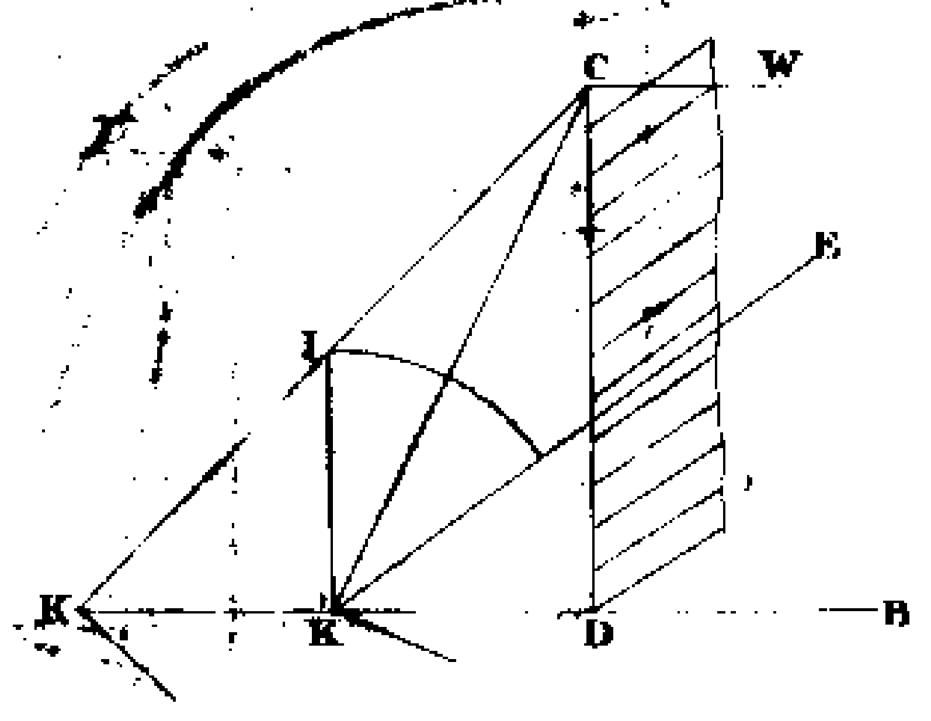

Fig. 13

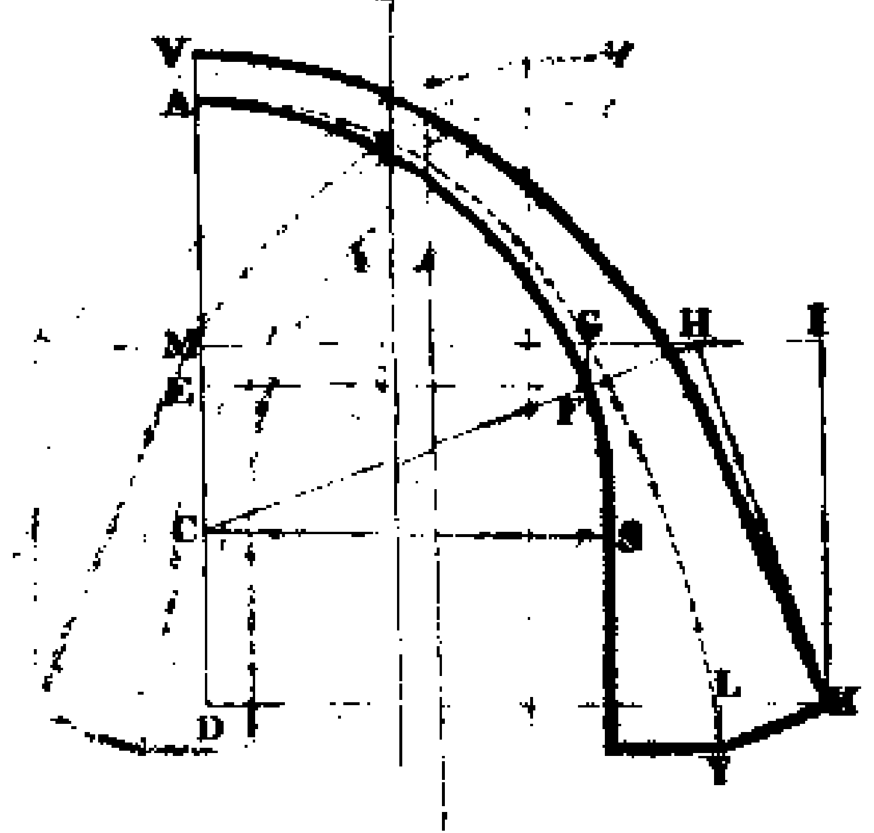

Fig. 14

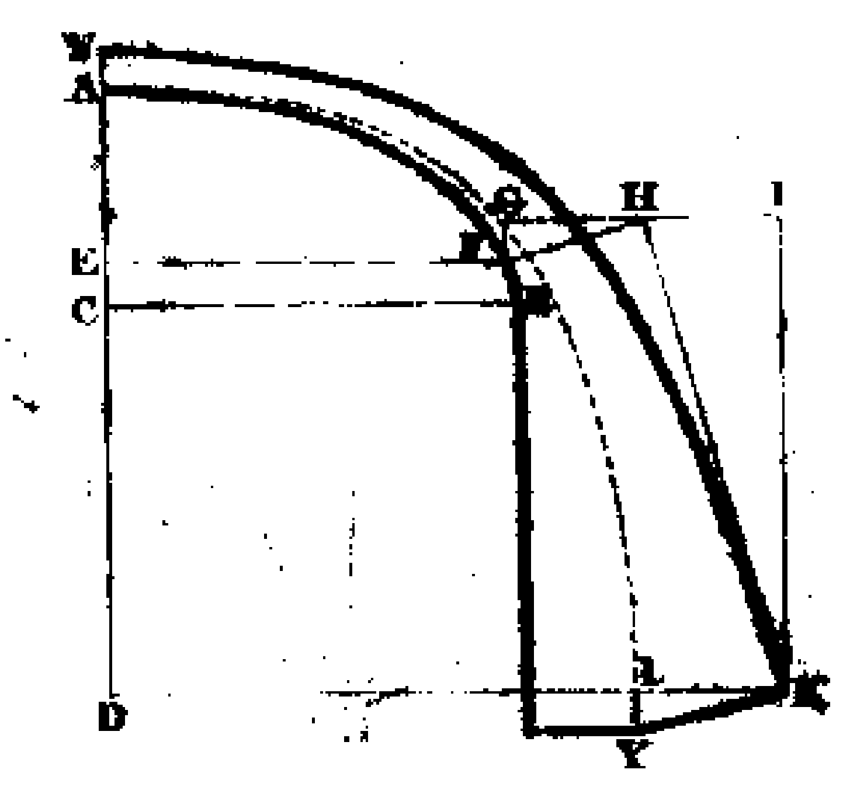

Fig. 15